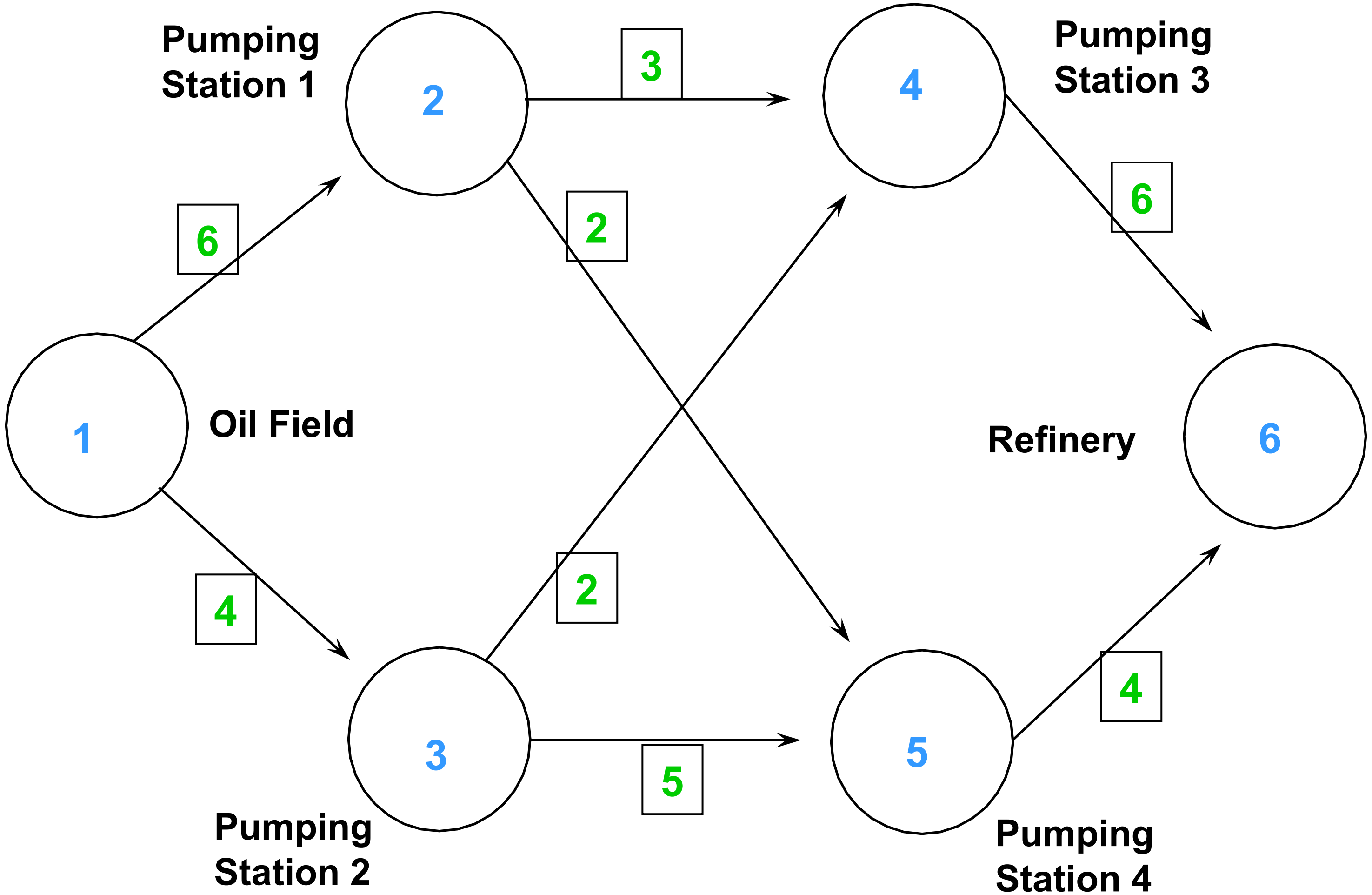
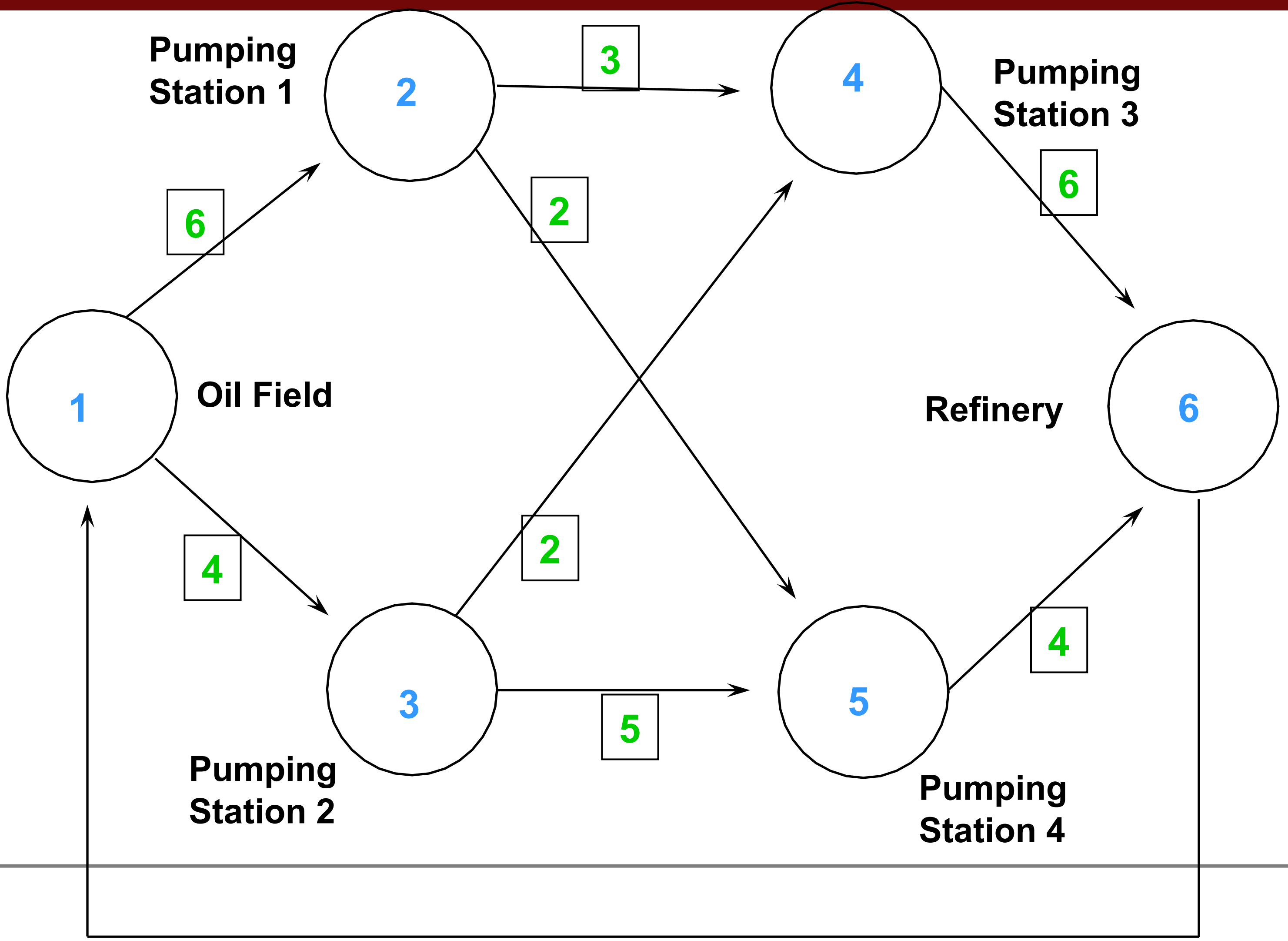
The Maximal Flow Problem

- In some network problems, the objective is to determine the maximum amount of flow that can occur through a network.
- The arcs in these problems have upper and lower flow limits.
- Examples
  - How much water can flow through a network of pipes?
  - How many cars can travel through a network of streets?

The Northwest Petroleum Company



The Northwest Petroleum Company



Formulation of the Max Flow Problem

MAX: X_{61}

Subject to:

$$+X_{61} - X_{12} - X_{13} = 0$$
$$+X_{12} - X_{24} - X_{25} = 0$$
$$+X_{13} - X_{34} - X_{35} = 0$$
$$+X_{24} + X_{34} - X_{46} = 0$$
$$+X_{25} + X_{35} - X_{56} = 0$$
$$+X_{46} + X_{56} - X_{61} = 0$$

with the following bounds on the decision variables:

$0 <= X_{12} <= 6$	$0 <= X_{25} <= 2$	$0 <= X_{46} <= 6$
$0 <= X_{13} <= 4$	$0 <= X_{34} <= 2$	$0 <= X_{56} <= 4$
$0 <= X_{24} <= 3$	$0 <= X_{35} <= 5$	$0 <= X_{61} <= inf$

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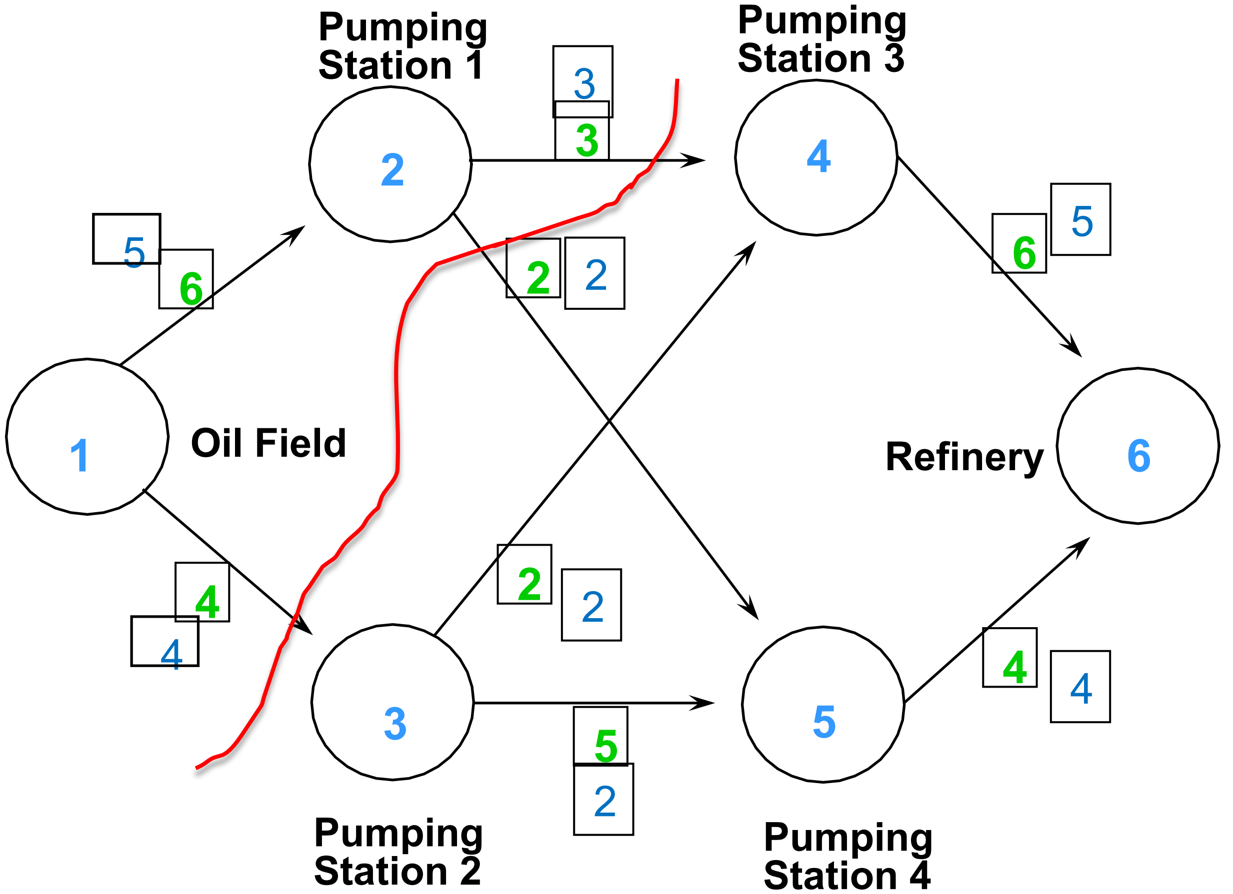
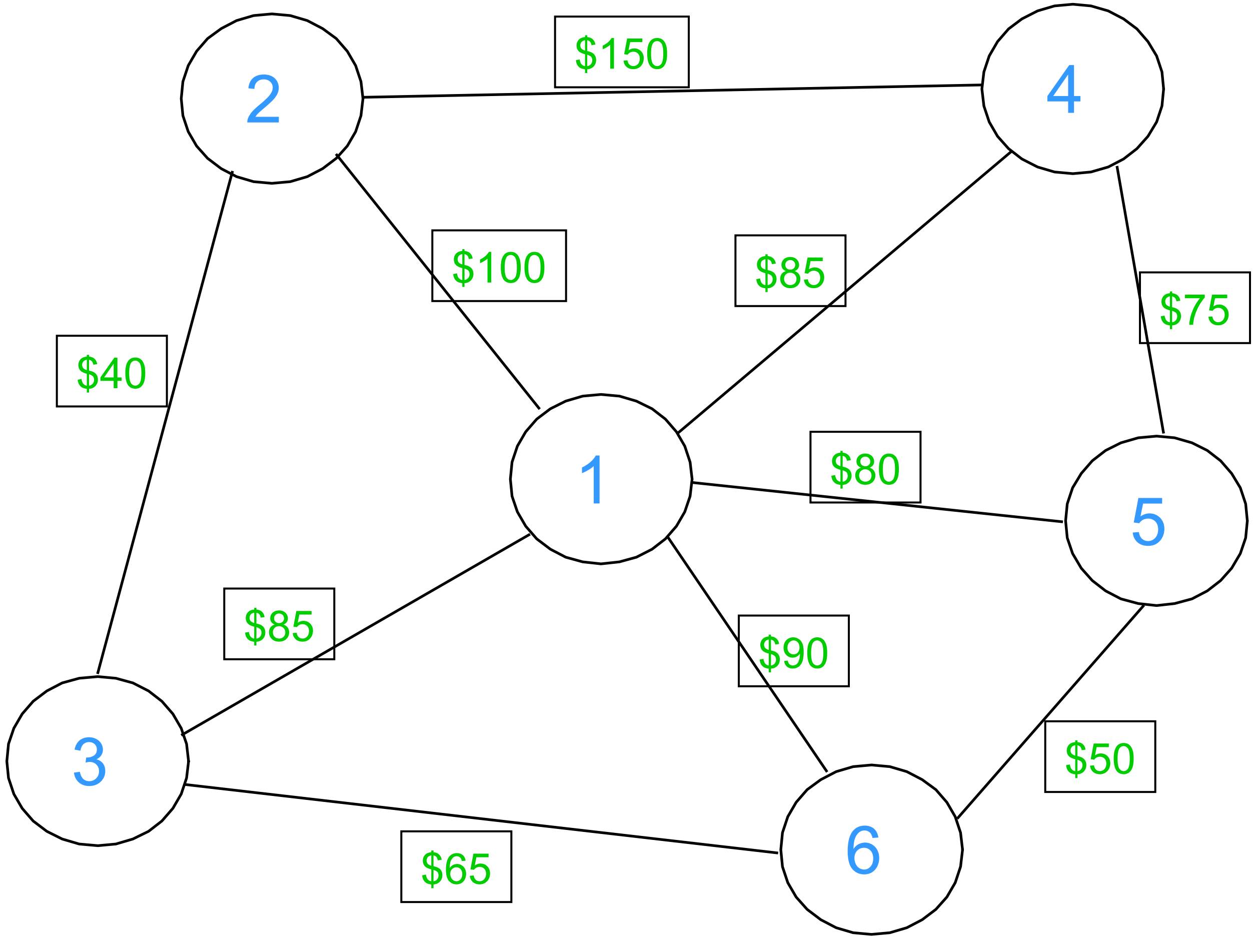
Optimal Solution



The Minimal Spanning Tree Problem

- For a network with n nodes, a *spanning tree* is a set of n-1 arcs that connects all the nodes and contains no loops.
- The minimal spanning tree problem involves determining the set of arcs that connects all the nodes at minimum cost.

Minimal Spanning Tree Example: Windstar Aerospace Company

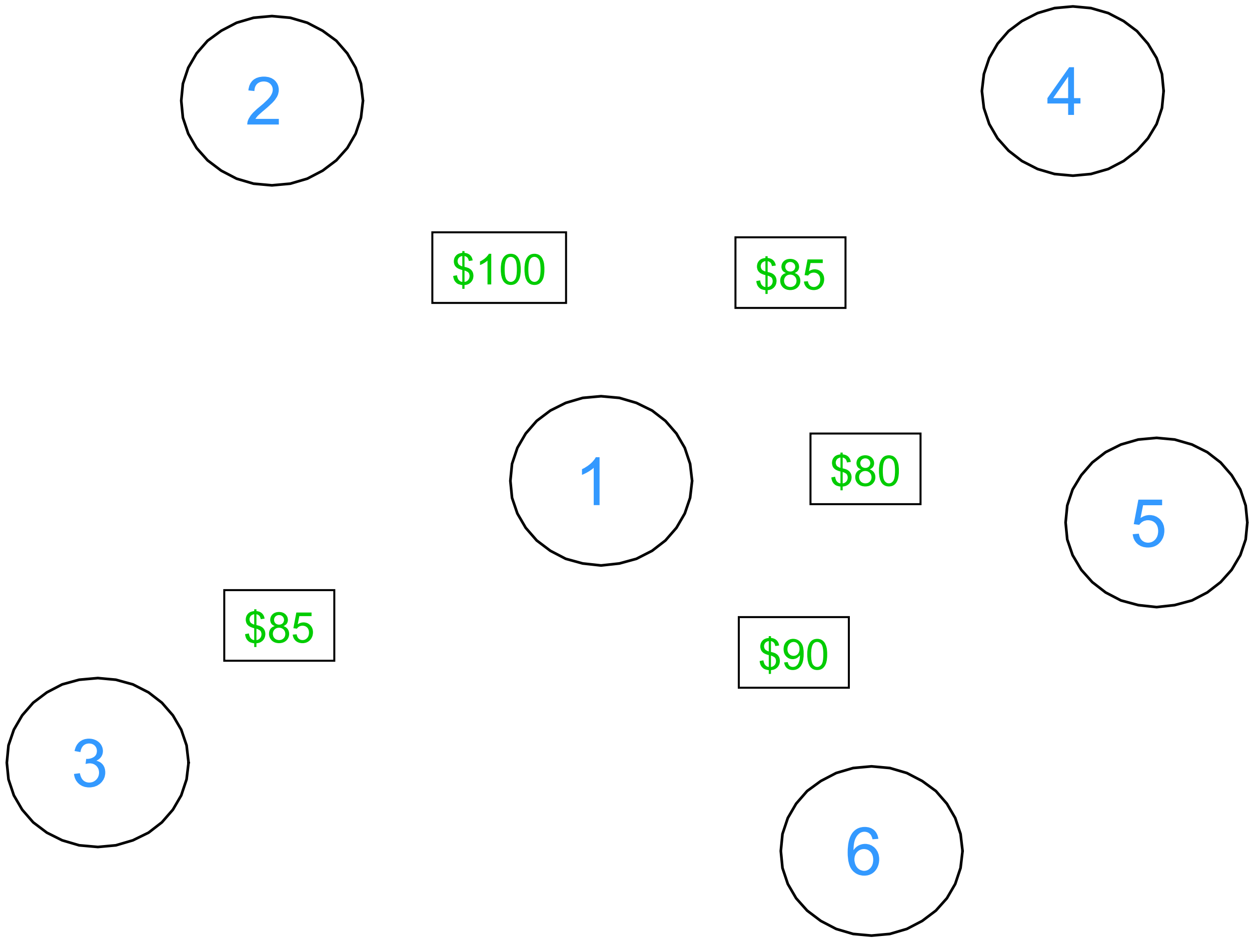


Nodes represent computers in a local area network.

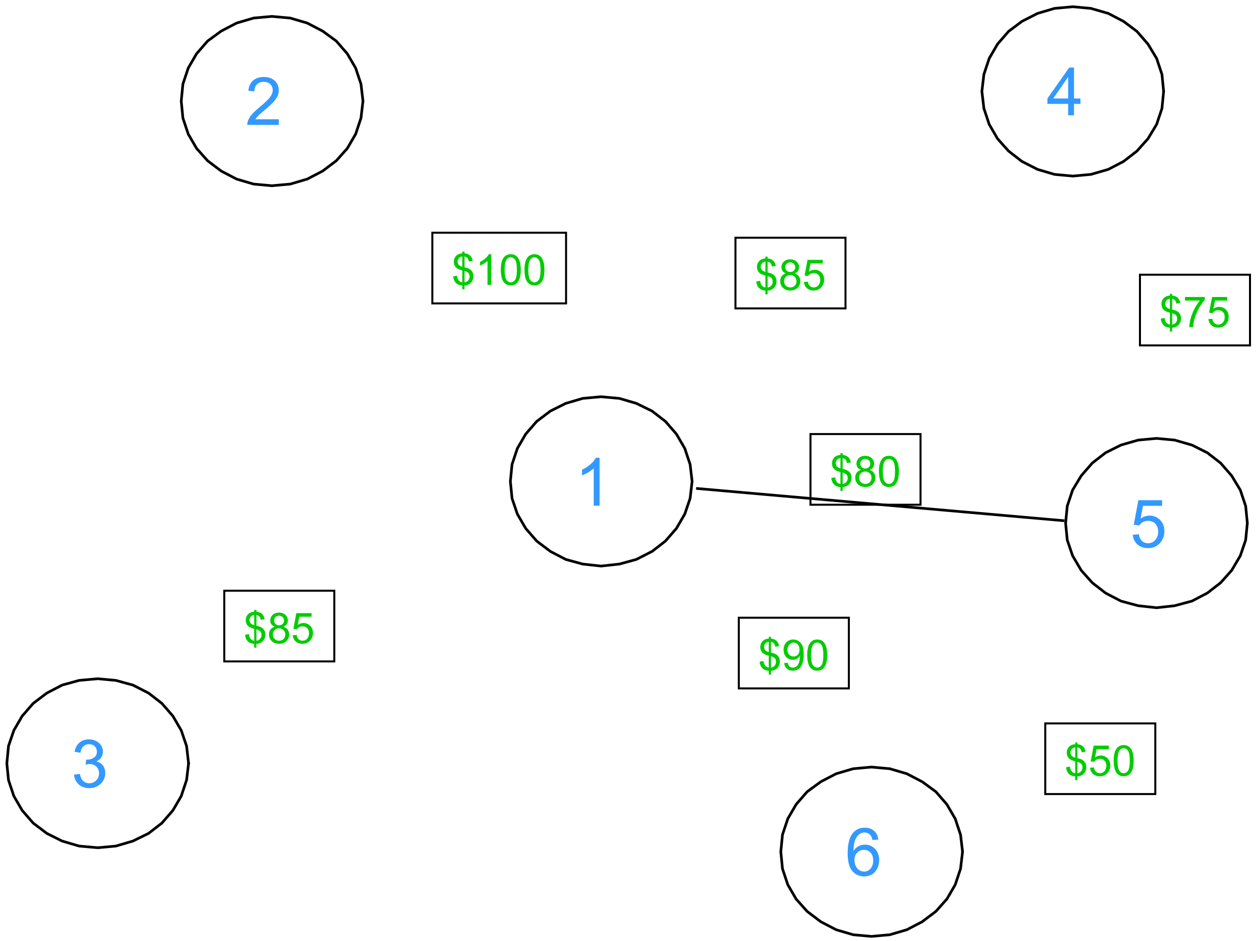
The Minimal Spanning Tree Algorithm

1. Select any node. Call this the current subnetwork.
2. Add to the current subnetwork the cheapest arc that connects any node within the current subnetwork to any node not in the current subnetwork. (Ties for the cheapest arc can be broken arbitrarily.) Call this the current subnetwork.
3. If all the nodes are in the subnetwork, stop; this is the optimal solution. Otherwise, return to step 2.

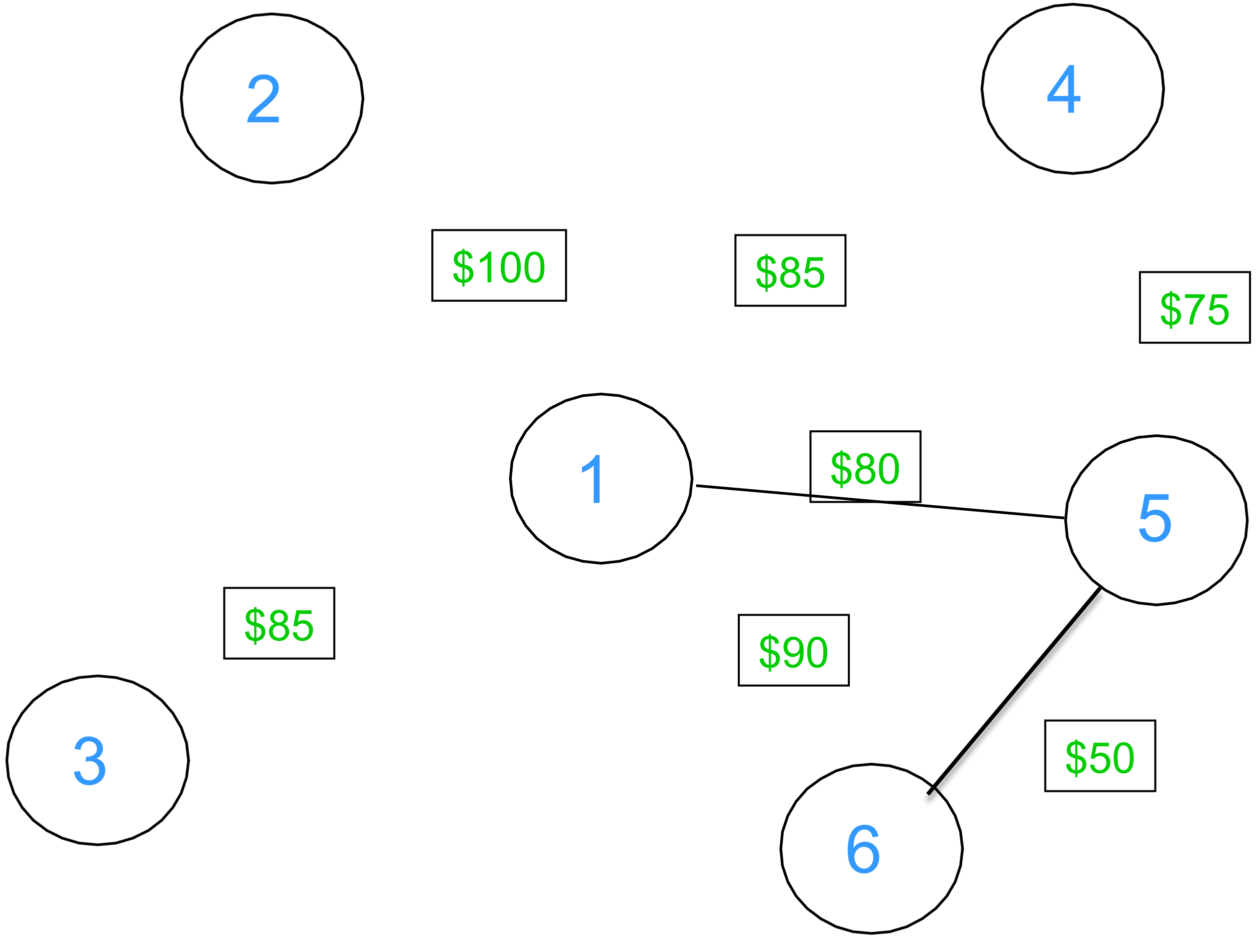
Solving the Example Problem - 1



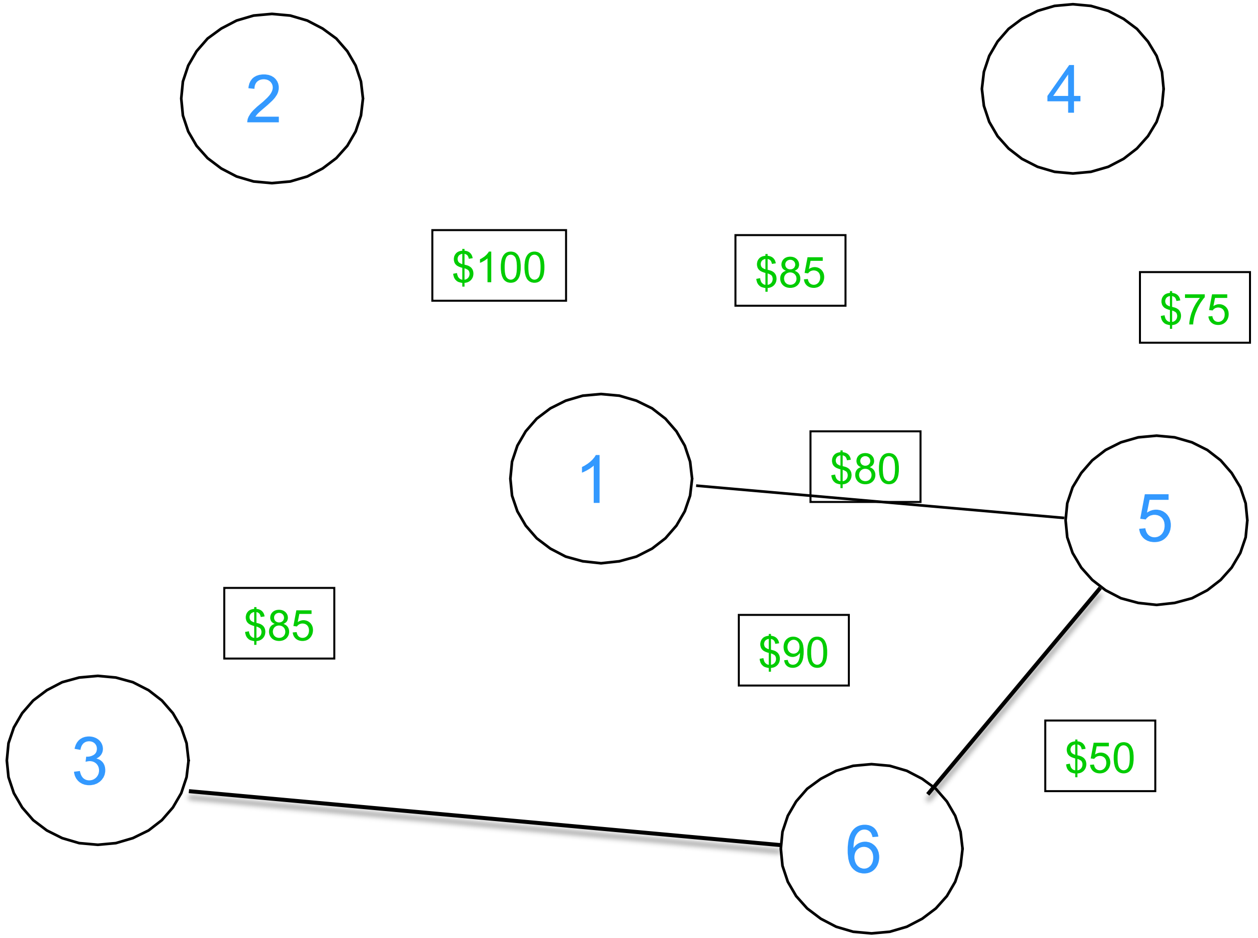
Solving the Example Problem - 2



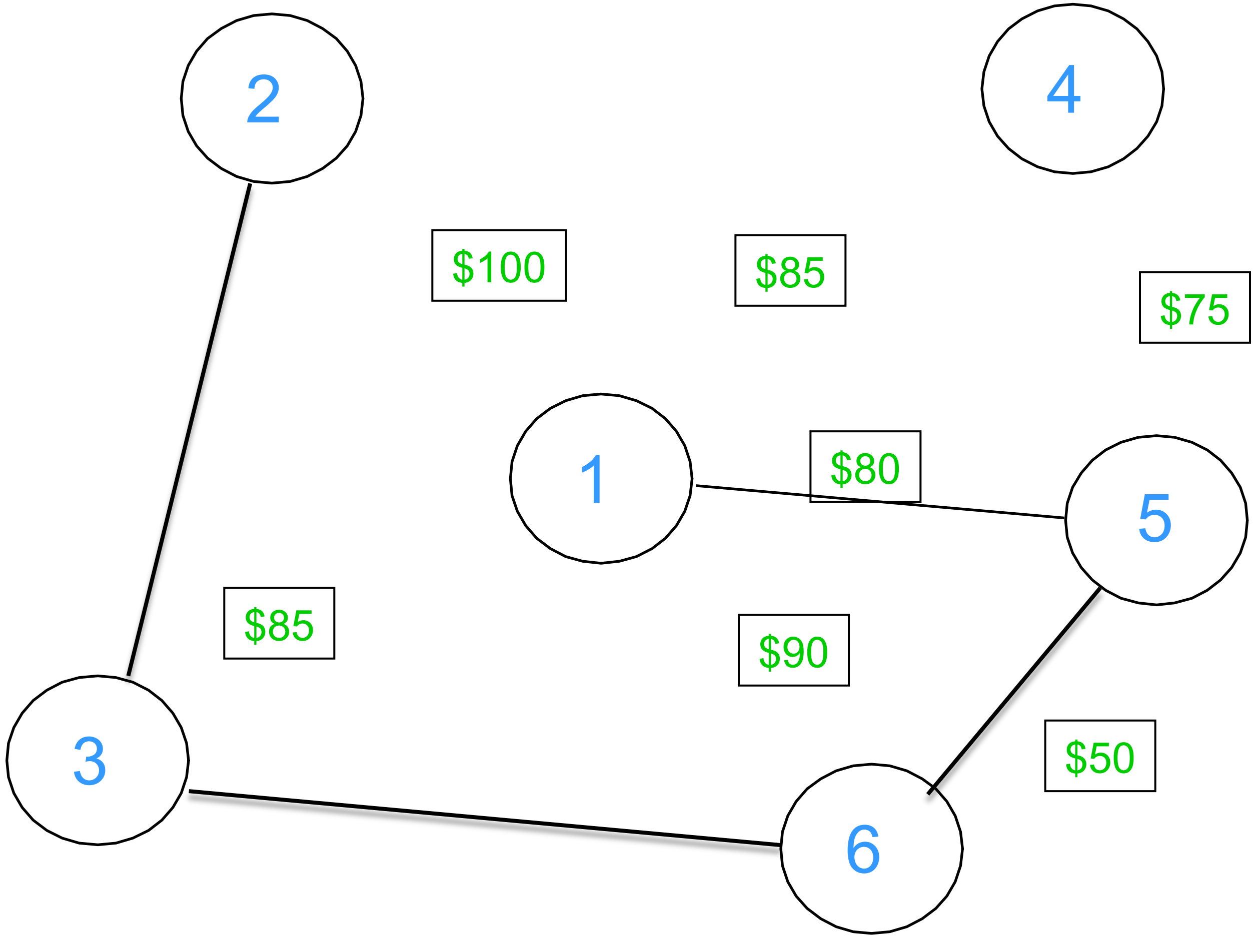
Solving the Example Problem - 2



Solving the Example Problem - 2



Solving the Example Problem - 2



Solving the Example Problem - 2