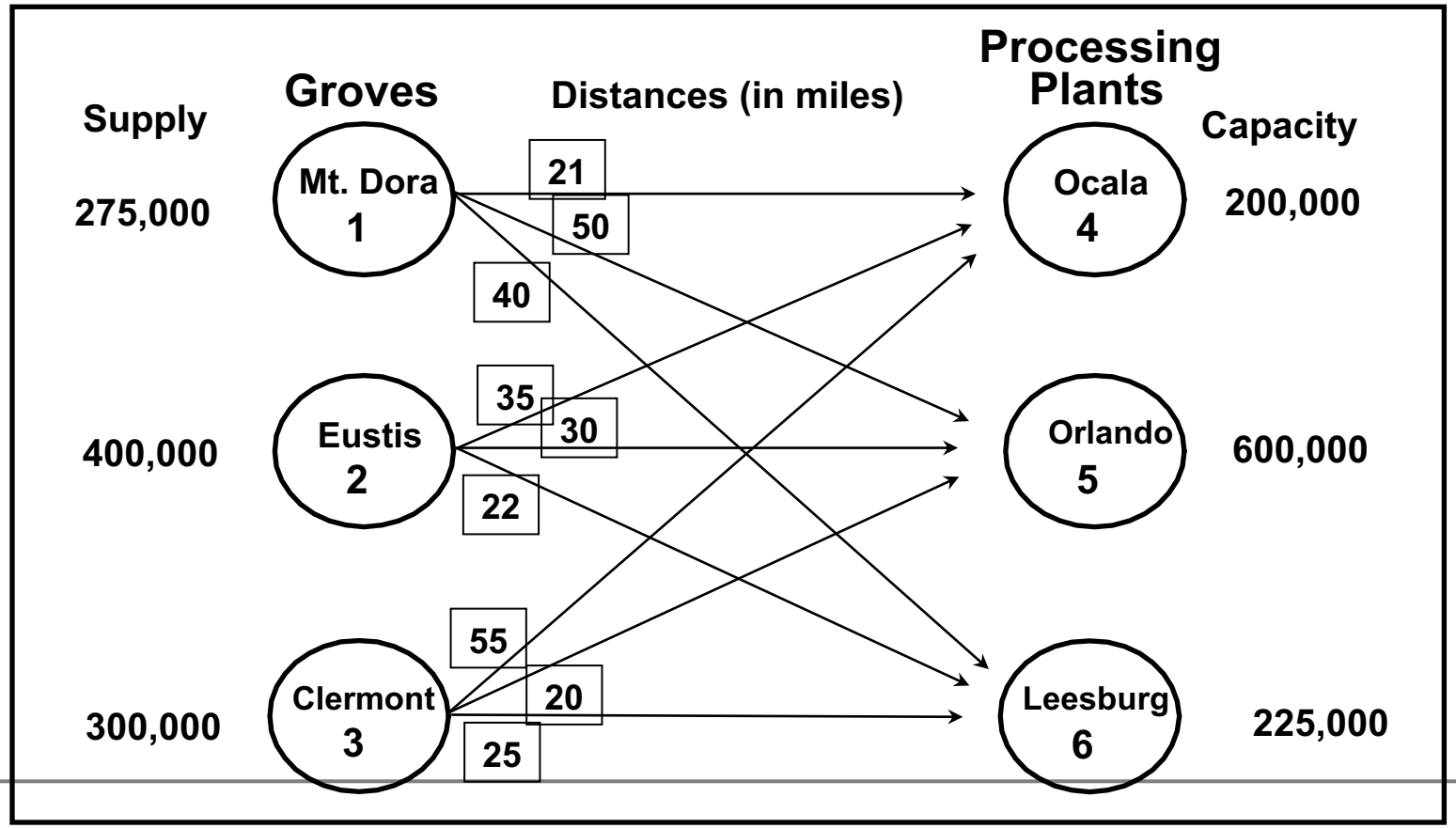
Transportation & Assignment Problems

- Some network flow problems don't have trans-shipment nodes; only supply and demand nodes.



Defining the Decision Variables

X_{ij} = # of bushels shipped from node i to node j

Specifically, the nine decision variables are:

X_{14} = # of bushels shipped from Mt. Dora (node 1) to Ocala (node 4)

X_{15} = # of bushels shipped from Mt. Dora (node 1) to Orlando (node 5)

X_{16} = # of bushels shipped from Mt. Dora (node 1) to Leesburg (node 6)

X_{24} = # of bushels shipped from Eustis (node 2) to Ocala (node 4)

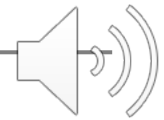
X_{25} = # of bushels shipped from Eustis (node 2) to Orlando (node 5)

X_{26} = # of bushels shipped from Eustis (node 2) to Leesburg (node 6)

X_{34} = # of bushels shipped from Clermont (node 3) to Ocala (node 4)

X_{35} = # of bushels shipped from Clermont (node 3) to Orlando (node 5)

X_{36} = # of bushels shipped from Clermont (node 3) to Leesburg (node 6)



Defining the Objective Function

Minimize the total number of bushel-miles.

$$\text{MIN: } 21X_{14} + 50X_{15} + 40X_{16} +$$
$$35X_{24} + 30X_{25} + 22X_{26} +$$
$$55X_{34} + 20X_{35} + 25X_{36}$$

Defining the Constraints

- Capacity constraints

$X_{14} + X_{24} + X_{34} <= 200{,}000$ } Ocala

$X_{15} + X_{25} + X_{35} <= 600{,}000$ } Orlando

$X_{16} + X_{26} + X_{36} <= 225{,}000$ } Leesburg

- Supply constraints

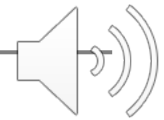
$X_{14} + X_{15} + X_{16} = 275{,}000$ } Mt. Dora

$X_{24} + X_{25} + X_{26} = 400{,}000$ } Eustis

$X_{34} + X_{35} + X_{36} = 300{,}000$ } Clermont

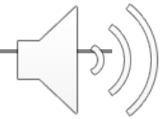
- Nonnegativity conditions

$X_{ij} >= 0$ for all i and j



Generalized Network Flow Problems

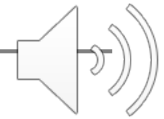
- In some problems, a gain or loss occurs in flows over arcs.
  - Examples
    - Oil or gas shipped through a leaky pipeline
    - Imperfections in raw materials entering a production process
    - Spoilage of food items during transit
    - Theft during transit
    - Interest or dividends on investments
- These problems require some modeling changes.



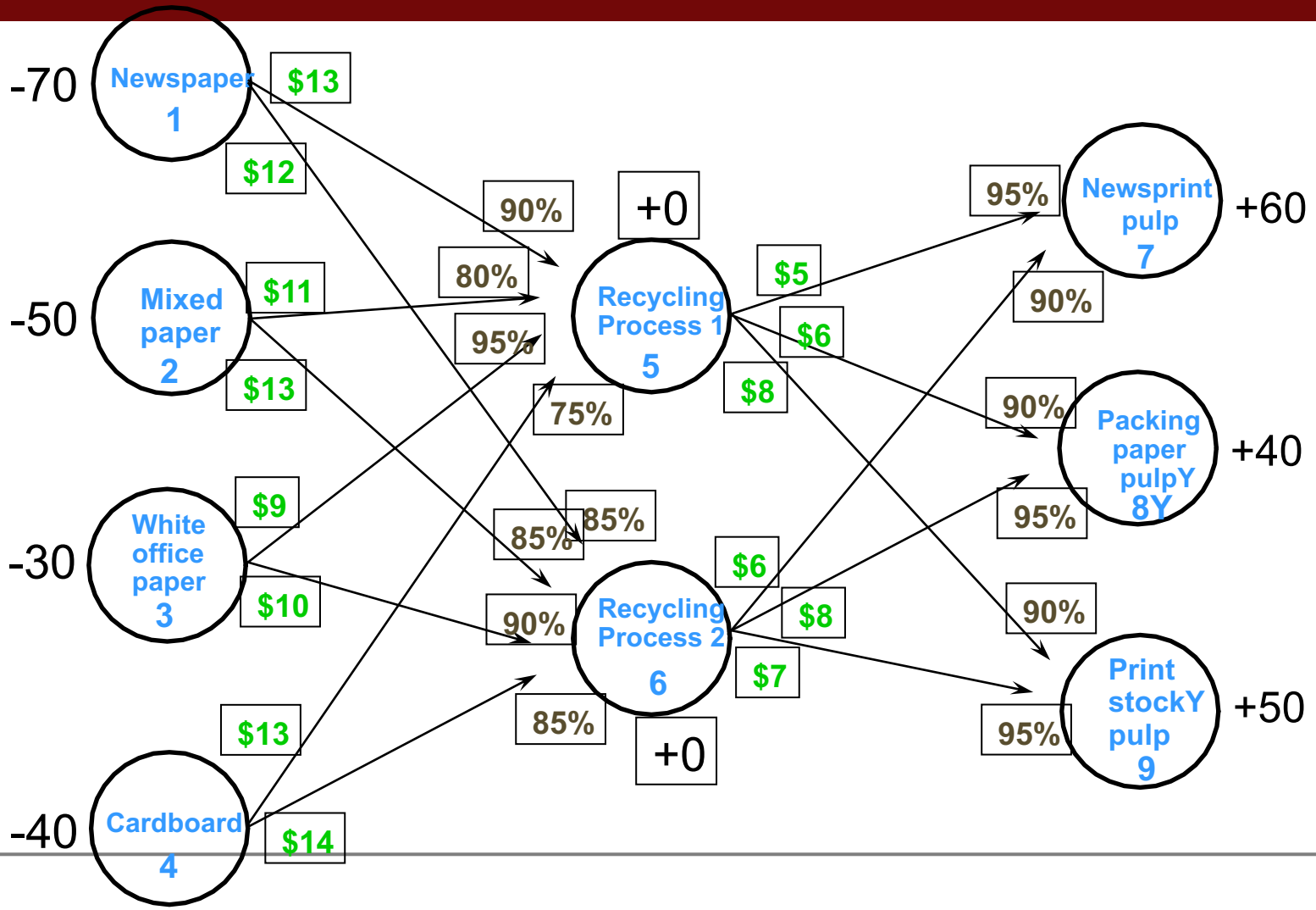
Coal Bank Hollow Recycling

	Process 1		Process 2		
Material	Cost	Yield	Cost	Yield	Supply
Newspaper	$13	90%	$12	85%	70 tons
Mixed Paper	$11	80%	$13	85%	50 tons
White Office Paper	$9	95%	$10	90%	30 tons
Cardboard	$13	75%	$14	85%	40 tons

	Newsprint		Packaging Paper		Print Stock	
Pulp Source	Cost	Yield	Cost	Yield	Cost	Yield
Recycling Process 1	$5	95%	$6	90%	$8	90%
Recycling Process 2	$6	90%	$8	95%	$7	95%
Demand	60 tons		40 tons		50 tons	



Network for Recycling Problem



Defining the Objective Function

Minimize total cost.

MIN: $13X_{15} + 12X_{16} + 11X_{25} + 13X_{26} + 9X_{35} + 10X_{36} + 13X_{45} +$

$+ 14X_{46} + 5X_{57} + 6X_{58} + 8X_{59} + 6X_{67} + 8X_{68} + 7X_{69}$

Defining the Constraints-I

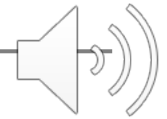
- Raw Materials

$-X_{15} - X_{16} >= -70$ } node 1

$-X_{25} - X_{26} >= -50$ } node 2

$-X_{35} - X_{36} >= -30$ } node 3

$-X_{45} - X_{46} >= -40$ } node 4



Defining the Constraints-II

- Recycling Processes

$$+0.9X_{15}+0.8X_{25}+0.95X_{35}+0.75X_{45}- X_{57}- X_{58}-X_{59} >= 0 \quad \} \text{ node 5}$$

$$+0.85X_{16}+0.85X_{26}+0.9X_{36}+0.85X_{46}-X_{67}-X_{68}-X_{69} >= 0 \quad \} \text{ node 6}$$

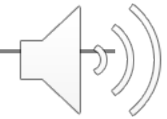
Defining the Constraints-III

- Paper Pulp

  $+0.95X_{57} + 0.90X_{67} >= 60$ } node 7

  $+0.90X_{57} + 0.95X_{67} >= 40$ } node 8

  $+0.90X_{57} + 0.95X_{67} >= 50$ } node 9



Problem Solution

Flow From Node			Yield	Flow Into Node			Cost
43.4	1	Newspaper	0.90	39.1	5	Process 1	$13
26.6	1	Newspaper	0.85	22.6	6	Process 2	$12
50.0	2	Mixed Paper	0.80	40.0	5	Process 1	$11
0.0	2	Mixed Paper	0.85	0.0	6	Process 2	$13
30.0	3	White Office	0.95	28.5	5	Process 1	$9
0.0	3	White Office	0.90	0.0	6	Process 2	$10
0.0	4	Cardboard	0.75	0.0	5	Process 1	$13
35.4	4	Cardboard	0.85	30.1	6	Process 2	$14
63.2	5	Process 1	0.95	60.0	7	Newsprint	$5
44.4	5	Process 1	0.90	40.0	8	Packaging	$6
0.0	5	Process 1	0.90	0.0	9	Print Stock	$8
0.0	6	Process 2	0.90	0.0	7	Newsprint	$6
0.0	6	Process 2	0.95	0.0	8	Packaging	$8
52.6	6	Process 2	0.95	50.0	9	Print Stock	$7
						Total Cost	$3,149