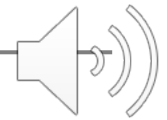
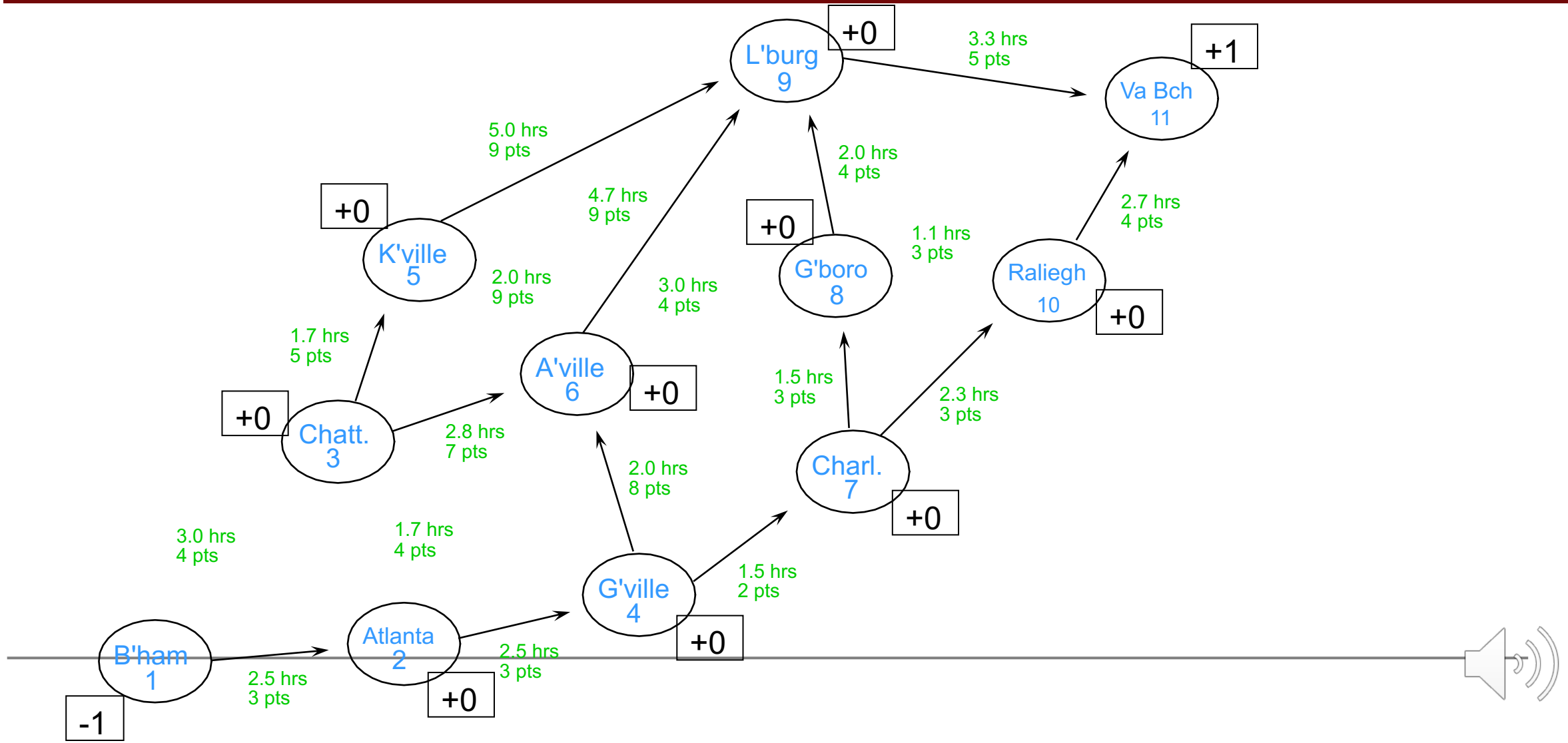
The Shortest Path Problem

- Many decision problems boil down to determining the shortest (or least costly) route or path through a network.
  - Ex. Vehicle Routing
- This is a special case of a transshipment problem where:
  - There is one supply node with a supply of -1
  - There is one demand node with a demand of +1
  - All other nodes have supply/demand of +0

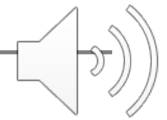


The American Car Association



Solving the Problem

- There are two possible objectives for this problem
  - Finding the quickest route (minimizing travel time)
  - Finding the most scenic route (maximizing the scenic rating points)

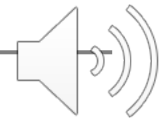


Solving the Problem

MIN: $+2.5X_{12} + 3X_{13} + 1.7X_{23} + 2.5X_{24} + 1.7X_{35} + 2.8X_{36} + 2X_{46} + 1.5X_{47} + 2X_{56} + 5X_{59}$
$+ 3X_{68} + 4.7X_{69} + 1.5X_{78} + 2.3X_{7,10} + 2X_{89} + 1.1X_{8,10} + 3.3X_{9,11} + 2.7X_{10,11}$

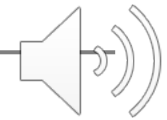
Subject to:

$-X_{12} - X_{13}$	$= -1$	} flow constraint for node 1
$+X_{12} - X_{23} - X_{24}$	$= 0$	} flow constraint for node 2
$+X_{13} + X_{23} - X_{35} - X_{36}$	$= 0$	} flow constraint for node 3
$+X_{24} - X_{46} - X_{47}$	$= 0$	} flow constraint for node 4
$+X_{35} - X_{56} - X_{59}$	$= 0$	} flow constraint for node 5
$+X_{36} + X_{46} + X_{56} - X_{68} - X_{69}$	$= 0$	} flow constraint for node 6
$+X_{47} - X_{78} - X_{7,10}$	$= 0$	} flow constraint for node 7
$+X_{68} + X_{78} - X_{89} - X_{8,10}$	$= 0$	} flow constraint for node 8
$+X_{59} + X_{69} + X_{89} - X_{9,11}$	$= 0$	} flow constraint for node 9
$+X_{7,10} + X_{8,10} - X_{10,11}$	$= 0$	} flow constraint for node 10
$+X_{9,11} + X_{10,11}$	$= +1$	} flow constraint for node 11
$X_{ij} \geq 0$ for all i and j		} nonnegativity conditions



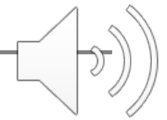
Solving the Problem

Select Route?	From		To		Driving Time	Scenic Rating
1.0	1	Birmingham	2	Atlanta	2.5	3
0.0	1	Birmingham	3	Chattanooga	3.0	4
0.0	2	Atlanta	3	Chattanooga	1.7	4
1.0	2	Atlanta	4	Greenville	2.5	3
0.0	3	Chattanooga	5	Knoxville	1.7	5
0.0	3	Chattanooga	6	Asheville	2.8	7
0.0	4	Greenville	6	Asheville	2.0	8
1.0	4	Greenville	7	Charlotte	1.5	2
0.0	5	Knoxville	6	Asheville	2.0	9
0.0	5	Knoxville	9	Lynchburg	5.0	9
0.0	6	Asheville	8	Greensboro	3.0	4
0.0	6	Asheville	9	Lynchburg	4.7	9
0.0	7	Charlotte	8	Greensboro	1.5	3
1.0	7	Charlotte	10	Raleigh	2.3	3
0.0	8	Greensboro	9	Lynchburg	2.0	4
0.0	8	Greensboro	10	Raleigh	1.1	3
0.0	9	Lynchburg	11	Virginia Beach	3.3	5
1.0	10	Raleigh	11	Virginia Beach	2.7	4
				Total	11.5	15



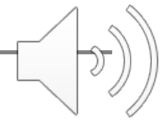
Solving the Problem

Select Route?	From		To		Driving Time	Scenic Rating
1.0	1	Birmingham	2	Atlanta	2.5	3
0.0	1	Birmingham	3	Chattanooga	3.0	4
1.0	2	Atlanta	3	Chattanooga	1.7	4
0.0	2	Atlanta	4	Greenville	2.5	3
1.0	3	Chattanooga	5	Knoxville	1.7	5
0.0	3	Chattanooga	6	Asheville	2.8	7
0.0	4	Greenville	6	Asheville	2.0	8
0.0	4	Greenville	7	Charlotte	1.5	2
1.0	5	Knoxville	6	Asheville	2.0	9
0.0	5	Knoxville	9	Lynchburg	5.0	9
0.0	6	Asheville	8	Greensboro	3.0	4
1.0	6	Asheville	9	Lynchburg	4.7	9
0.0	7	Charlotte	8	Greensboro	1.5	3
0.0	7	Charlotte	10	Raleigh	2.3	3
0.0	8	Greensboro	9	Lynchburg	2.0	4
0.0	8	Greensboro	10	Raleigh	1.1	3
1.0	9	Lynchburg	11	Virginia Beach	3.3	5
0.0	10	Raleigh	11	Virginia Beach	2.7	4
				Total	15.9	35



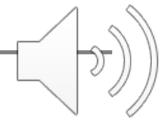
The Equipment Replacement Problem

- The problem of determining when to replace equipment is another common business problem.
- It can also be modeled as a shortest path problem…

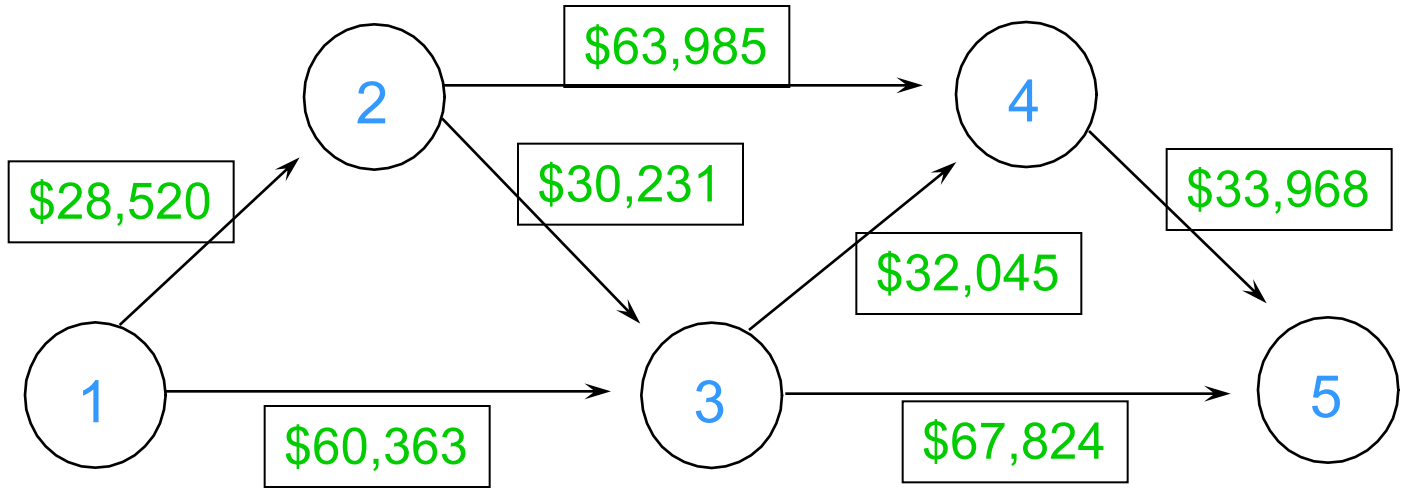


The Compu-Train Company

- Compu-Train provides hands-on software training.
- Computers must be replaced at least every two years.
- Two lease contracts are being considered:
  - Each requires $62,000 initially
  - Contract 1:
    - Prices increase 6% per year
    - 60% trade-in for 1 year old equipment
    - 15% trade-in for 2 year old equipment
  - Contract 2:
    - Prices increase 2% per year
    - 30% trade-in for 1 year old equipment
    - 10% trade-in for 2 year old equipment



Network for Contract 1



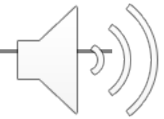
Cost of trading after 1 year: $1.06*\$62,000 - 0.6*\$62,000 = \$28,520$

Cost of trading after 2 years: $1.06^2*\$62,000 - 0.15*\$62,000 = \$60,363$

etc, etc….

Solving the Problem

Select	From	To	Cost
1.0	1	2	$28,520
0.0	1	3	$60,363
1.0	2	3	$30,231
0.0	2	4	$63,985
1.0	3	4	$32,045
0.0	3	5	$67,824
1.0	4	5	$33,968
	Total Cost		$124,764



Solving the Problem

Select	From	To	Cost
1.0	1	2	$28,520
0.0	1	3	$60,363
1.0	2	3	$30,231
0.0	2	4	$63,985
1.0	3	4	$32,045
0.0	3	5	$67,824
1.0	4	5	$33,968
		Total Cost	$124,764

Select	From	To	Cost
0.0	1	2	$44,640
1.0	1	3	$58,305
0.0	2	3	$45,533
0.0	2	4	$59,471
0.0	3	4	$46,443
1.0	3	5	$60,660
0.0	4	5	$47,372
		Total Cost	$118,965