Network Models

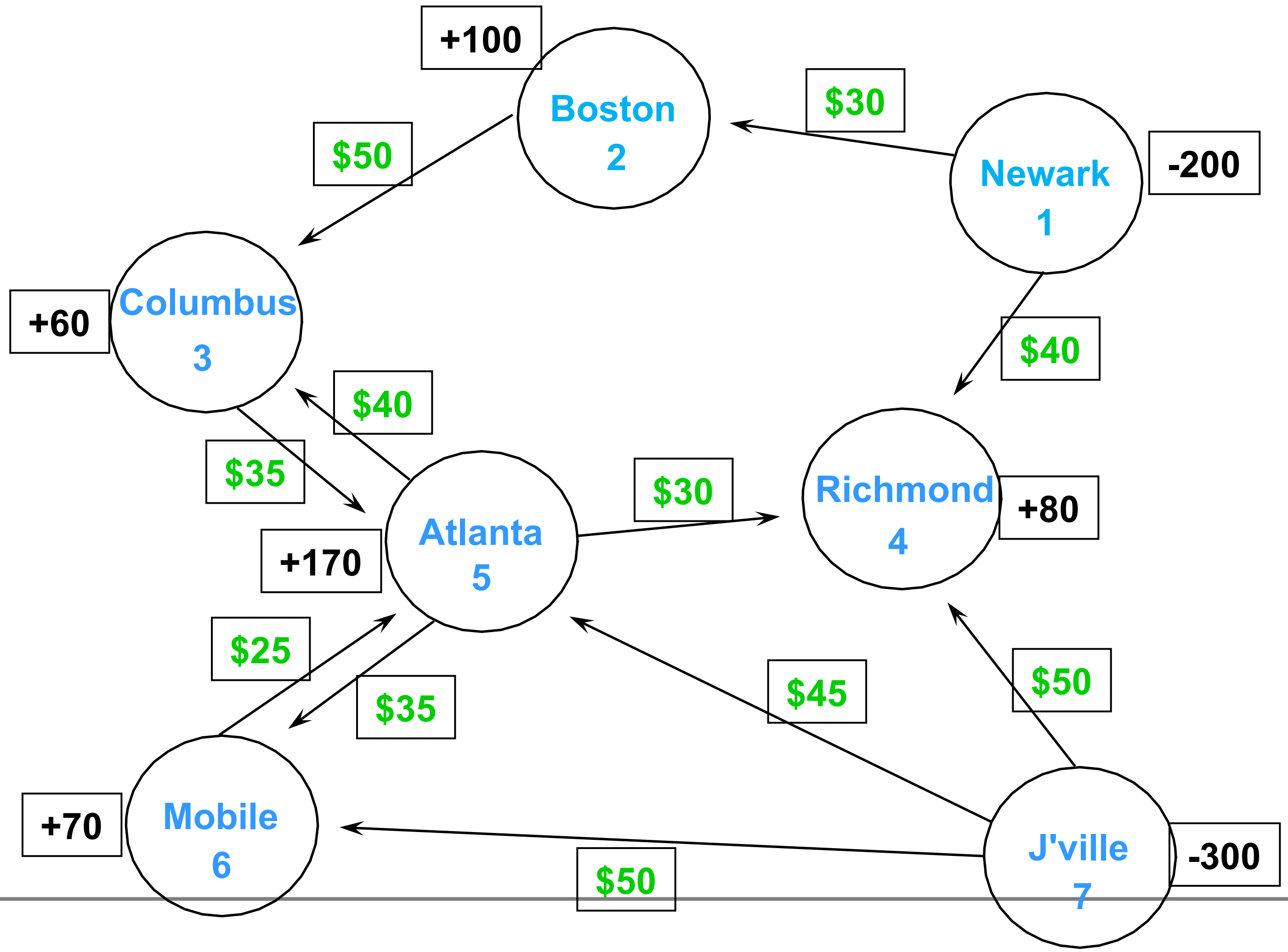
Introduction

- A number of business problems can be represented graphically as networks.
- This chapter focuses on several such problems:
    - Transshipment Problems
    - Shortest Path Problems
    - Maximal Flow Problems
    - Transportation/Assignment Problems
    - Generalized Network Flow Problems
    - The Minimum Spanning Tree Problem

Network Flow Problem Characteristics

- Network flow problems can be represented as a collection of nodes connected by arcs.
- There are three types of nodes:
  - Supply
  - Demand
  - Transshipment
- We'll use negative numbers to represent supplies and positive numbers to represent demand.

A Transshipment Problem: The Bavarian Motor Company



Defining the Decision Variables

For each arc in a network flow model
we define a decision variable as:

X_{ij} = the amount being shipped (or flowing) *from* node i *to* node j

For example…

X_{12} = the # of cars shipped *from* node 1 (Newark) *to* node 2 (Boston)

X_{56} = the # of cars shipped *from* node 5 (Atlanta) *to* node 6 (Mobile)

Note: The number of arcs determines
the number of variables!

Defining the Objective Function

Minimize total shipping costs.

$$\text{MIN: } 30X_{12} + 40X_{14} + 50X_{23} + 35X_{35}$$
$$+ 40X_{53} + 30X_{54} + 35X_{56} + 25X_{65}$$
$$+ 50X_{74} + 45X_{75} + 50X_{76}$$

Constraints for Network Flow Problems: The Balance-of-Flow Rules

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule At Each Node:
Total Supply > Total Demand	Inflow-Outflow >= Supply or Demand
Total Supply < Total Demand	Inflow-Outflow <=Supply or Demand
Total Supply = Total Demand	Inflow-Outflow = Supply or Demand

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Defining the Constraints

- In the BMC problem:

  Total Supply = 500 cars

  Total Demand = 480 cars

  (Supply >= Demand)

- For each node we need a constraint like this:

  Inflow - Outflow >= Supply or Demand

- Constraint for node 1:

  $-X_{12} - X_{14} >= -200$ (Note: there is no inflow for node 1!)

- This is equivalent to:

  $+X_{12} + X_{14} <= 200$

Defining the Constraints

- Flow constraints

$-X_{12} - X_{14} >= -200$ } node 1

$+X_{12} - X_{23} >= +100$ } node 2

$+X_{23} + X_{53} - X_{35} >= +60$ } node 3

$+ X_{14} + X_{54} + X_{74} >= +80$ } node 4

$+ X_{35} + X_{65} + X_{75} - X_{53} - X_{54} - X_{56} >= +170$ } node 5

$+ X_{56} + X_{76} - X_{65} >= +70$ } node 6

$-X_{74} - X_{75} - X_{76} >= -300$ } node 7

- Nonnegativity conditions

$X_{ij} >= 0$ for all ij

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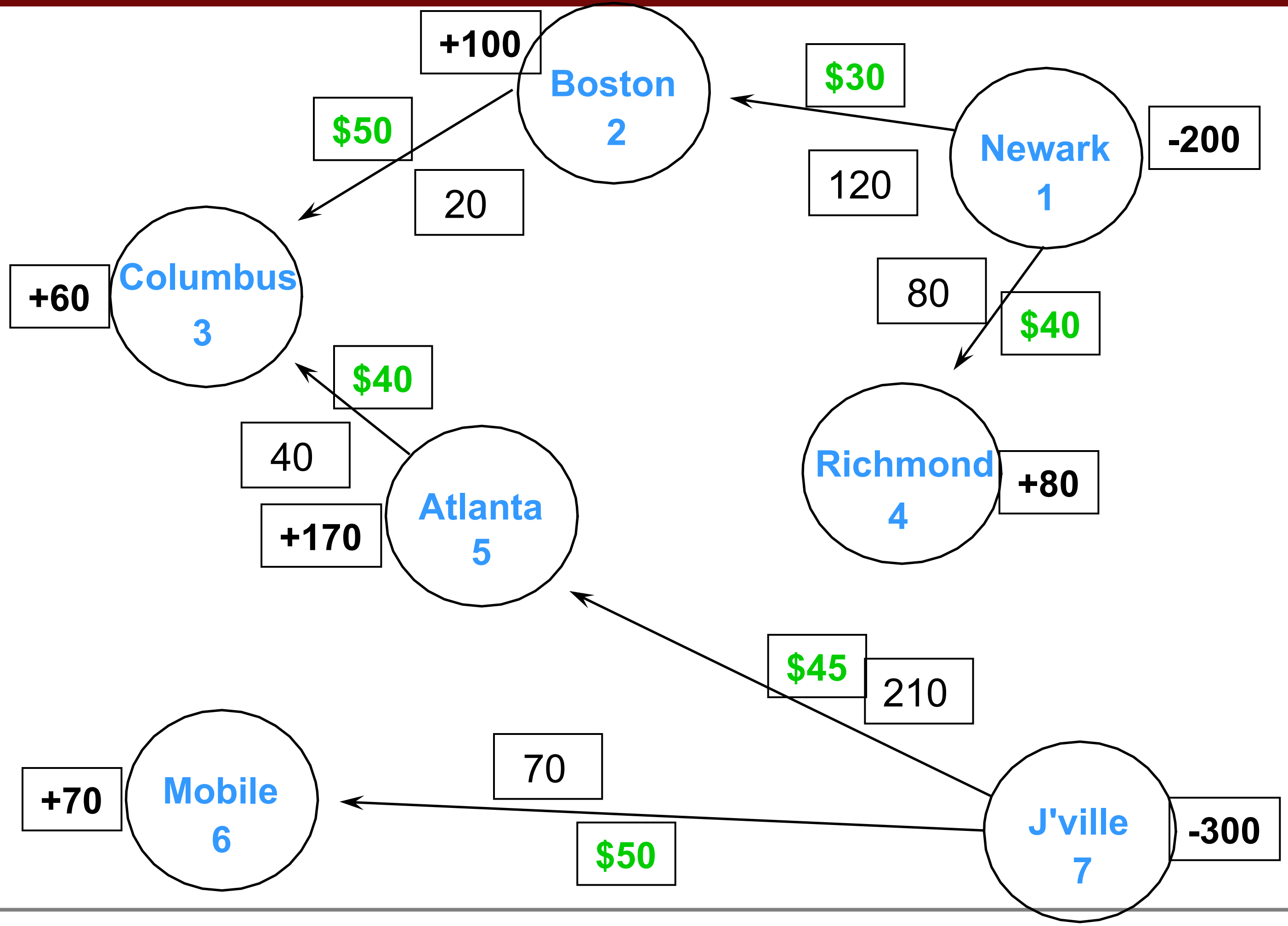
Optimal Solution to the BMC Problem



Total cost = $22,350