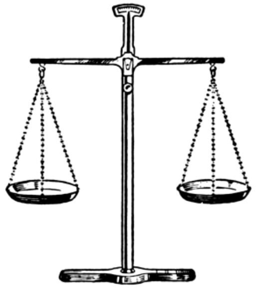
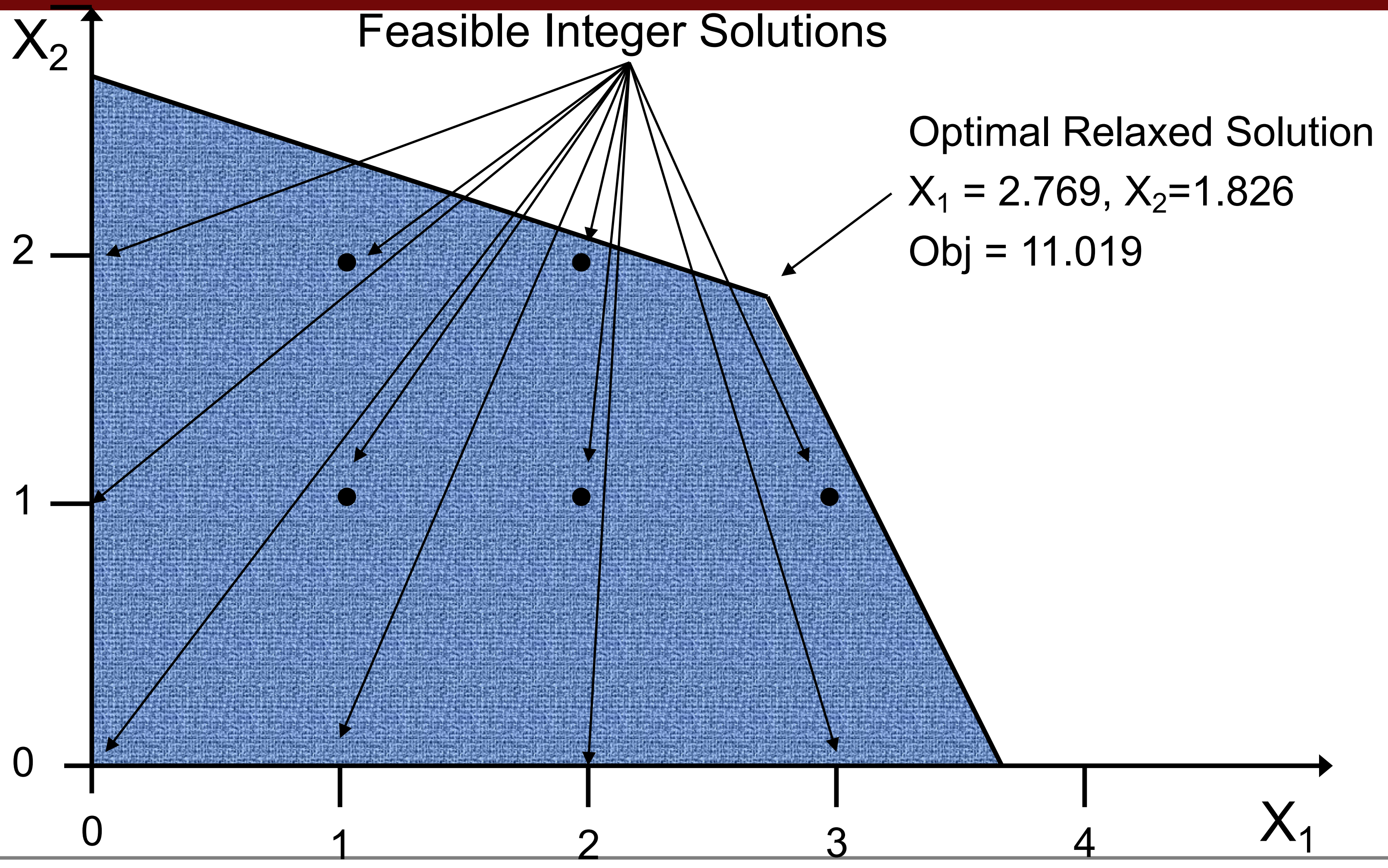


Branch and Bound Algorithm

The Branch-And-Bound Algorithm

$$
\begin{aligned}
\text{MAX:}\quad & 2X_1 + 3X_2 \\
\text{S.T.}\quad & X_1 + 3X_2 <= 8.25 \\
& 2.5X_1 + X_2 <= 8.75 \\
& X_1, X_2 >= 0 \text{ and integer}
\end{aligned}
$$

Solution to LP Relaxation



The Branch-And-Bound Algorithm

Problem I MAX: $2X_1 + 3X_2$

S.T. $X_1 + 3X_2 <= 8.25$

$2.5X_1 + X_2 <= 8.75$

$X_1 <= 2$

$X_1, X_2 >= 0$ and integer

Problem II MAX: $2X_1 + 3X_2$

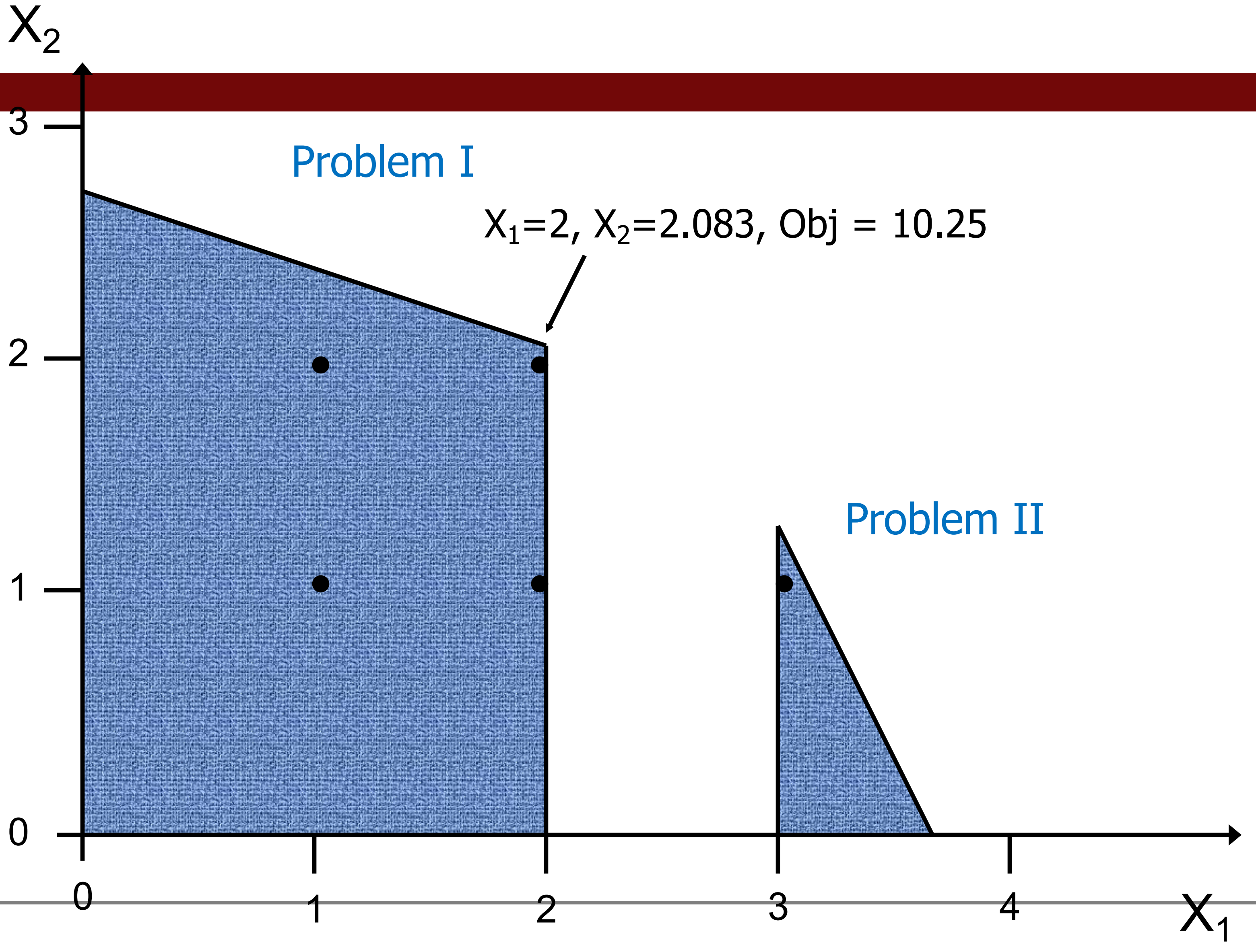
S.T. $X_1 + 3X_2 <= 8.25$

$2.5X_1 + X_2 <= 8.75$

$X_1 >= 3$

$X_1, X_2 >= 0$ and integer

Solution to LP Relaxation



The Branch-And-Bound Algorithm

Problem III

MAX: $2X_1 + 3X_2$

S.T. $X_1 + 3X_2 <= 8.25$

$2.5X_1 + X_2 <= 8.75$

$X_1 <= 2$

$X_2 <= 2$

$X_1, X_2 >= 0$ and integer

Problem IV

MAX: $2X_1 + 3X_2$

S.T. $X_1 + 3X_2 <= 8.25$

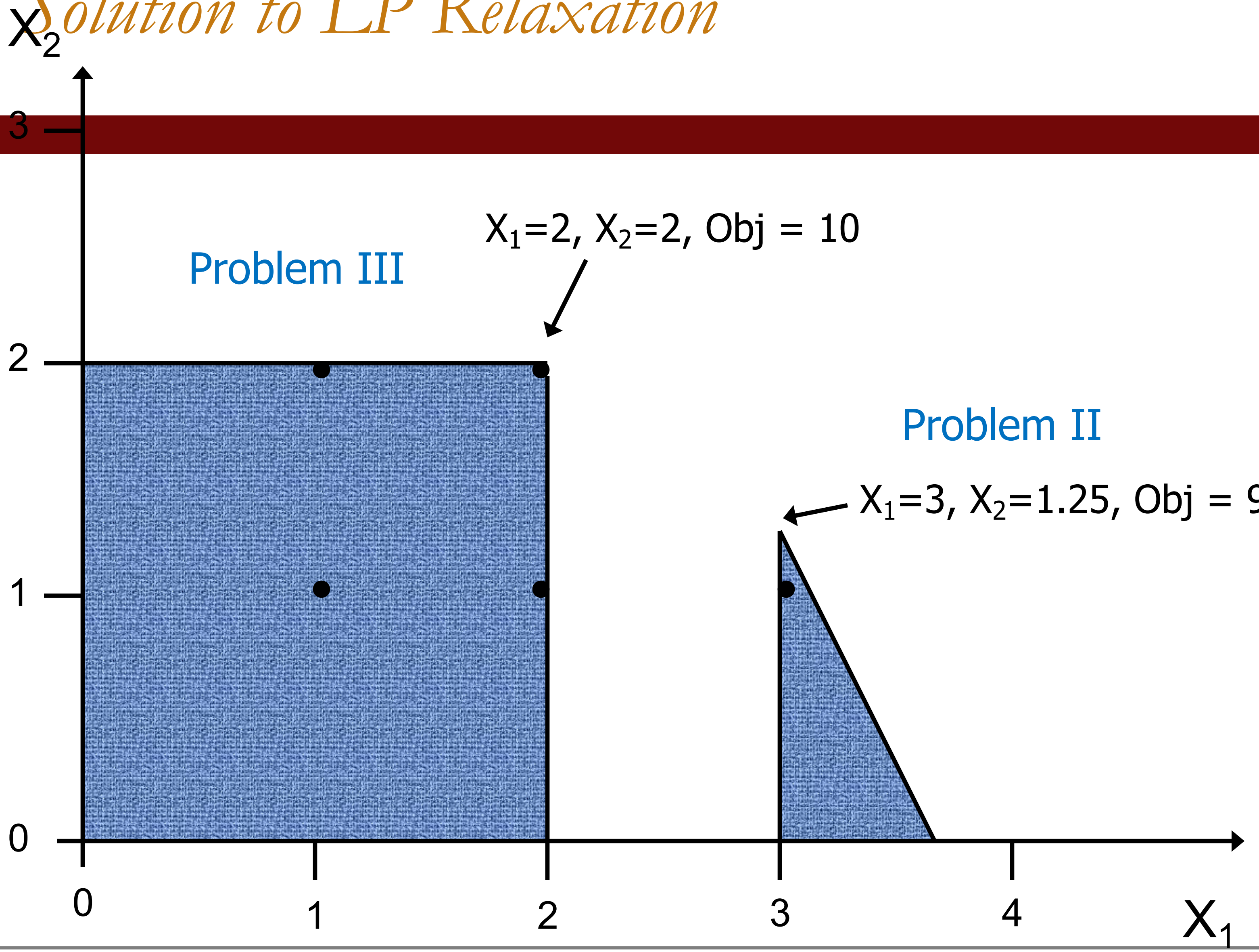
$2.5X_1 + X_2 <= 8.75$

$X_1 <= 2$

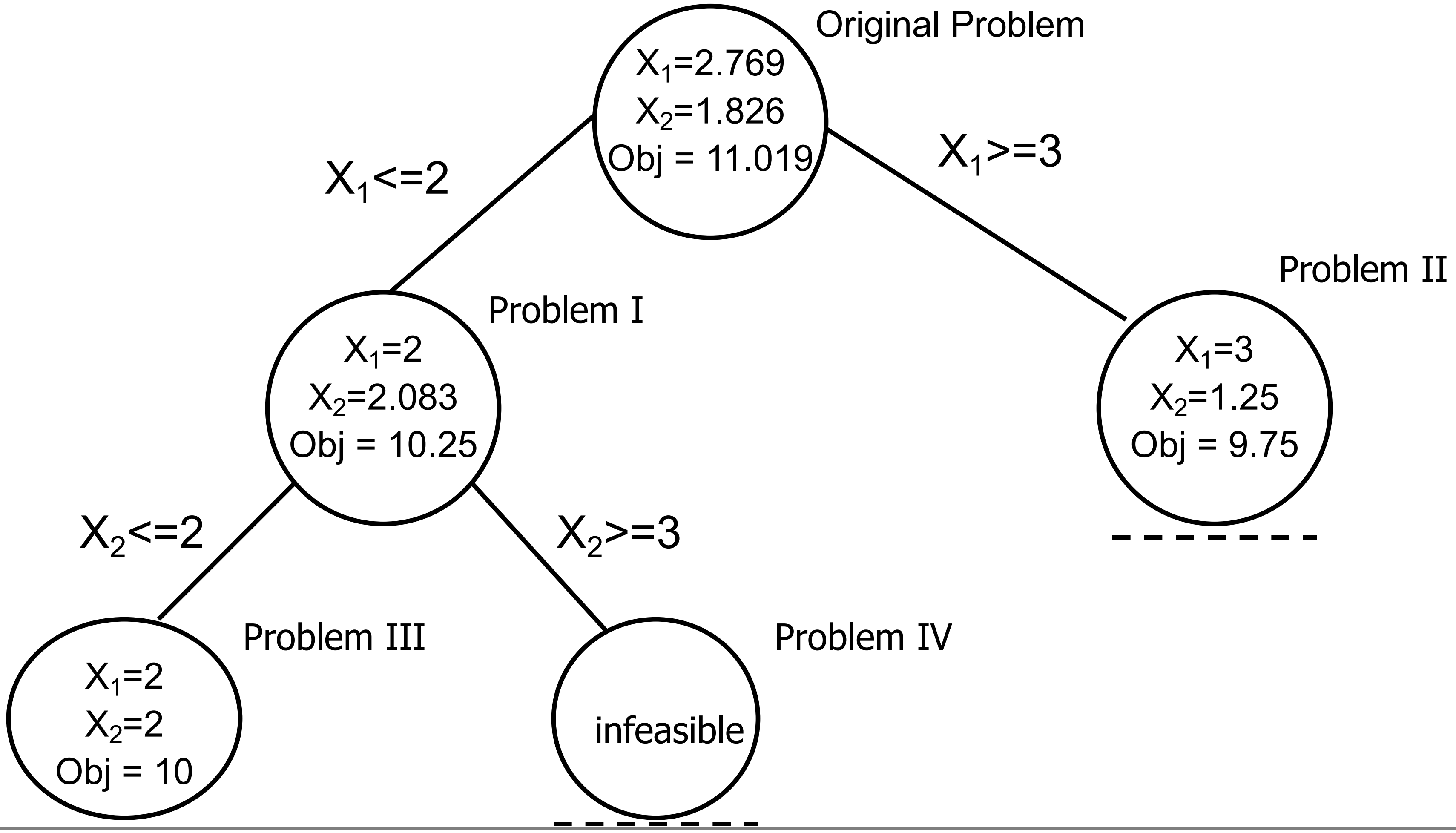
$X_2 >= 3$

$X_1, X_2 >= 0$ and integer

Solution to LP Relaxation



B&B Summary


Original Problem
X1=2.769
X2=1.826
Obj = 11.019
X1<=2
X1>=3
Problem I
X1=2
X2=2.083
Obj = 10.25
Problem II
X1=3
X2=1.25
Obj = 9.75
X2<=2
X2>=3
Problem III
X1=2
X2=2
Obj = 10
Problem IV
infeasible

The Branch and Bound Algorithm

INITIALIZATION

- Relax all the integrality conditions in ILP and solve the resulting LP problem.

If the optimal solution to the relaxed LP problem happens to satisfy the original integrality conditions, stop—this is the optimal integer solution.

Otherwise, proceed to step 2.

- If the problem being solved is a maximization problem let $Z_{best} = -\infty$. If it is a minimization problem, let $Z_{best} = \infty$.

In general Z_{best} represents the objective function value of the best known integer solution as the algorithm proceeds.

The Branch and Bound Algorithm

2. BRANCHING

Let X_j represents one of the variables that violated the integrality conditions in the optimal solution to the problem that was solved most recently and let b_j represent its non-integer value.

Let INT(b_j) represent the largest integer that is less than b_j.

Create two new candidate problems: one by appending the constraint $Xj \leq$ INT(b_j) to the most recently solved LP problem, and the other by appending the constraint $Xj \geq$ INT(bj) + 1 to the most recently solved LP problem.

Place both of these new LP problems in a list of candidate problems to be solved.

The Branch and Bound Algorithm

3. SOLVE THE RELAXED SUBPROBLEMS AND BOUNDING

a. If the list of candidate problems is empty, proceed to step 6 (STOP). Otherwise, remove a candidate problem from the list, relax any integrality conditions in the problem, and solve it.

b. If there is not a solution to the current candidate problem (that is, it is infeasible), proceed to step 3.a. Otherwise, let Z_{cp} denote the optimal objective function value for the current candidate problem.

c. If Z_{cp} is not better than Z_{best} (for a maximization problem $Z_{cp} \leq Z_{best}$ or for a minimization problem $Z_{cp} \geq Z_{best}$), proceed to step 3.a.

.

The Branch and Bound Algorithm

d. If the solution to the current candidate problem does not satisfy the original integrality conditions, and Z_{cp} is better than Z_{best} proceed to step 2 (BRANCHING).

e. If the solution to the current candidate problem does satisfy the original integrality conditions, a better integer solution has been found.

Thus, let $Z_{best} = Z_{cp}$ and save the solution obtained for this candidate problem.

Then go back to step 3.

4. STOP. The optimal solution has been found and has an objective function value given by the current value of Z_{best}.

The Branch and Bound Algorithm

d. If the solution to the current candidate problem does not satisfy the original integrality conditions, and Z_{cp} is better than Z_{best} proceed to step 2 (BRANCHING).

e. If the solution to the current candidate problem does satisfy the original integrality conditions, a better integer solution has been found.
Thus, let $Z_{best} = Z_{cp}$ and save the solution obtained for this candidate problem.
Then go back to step 3.

4. STOP. The optimal solution has been found and has an objective function value given by the current value of Z_{best}.