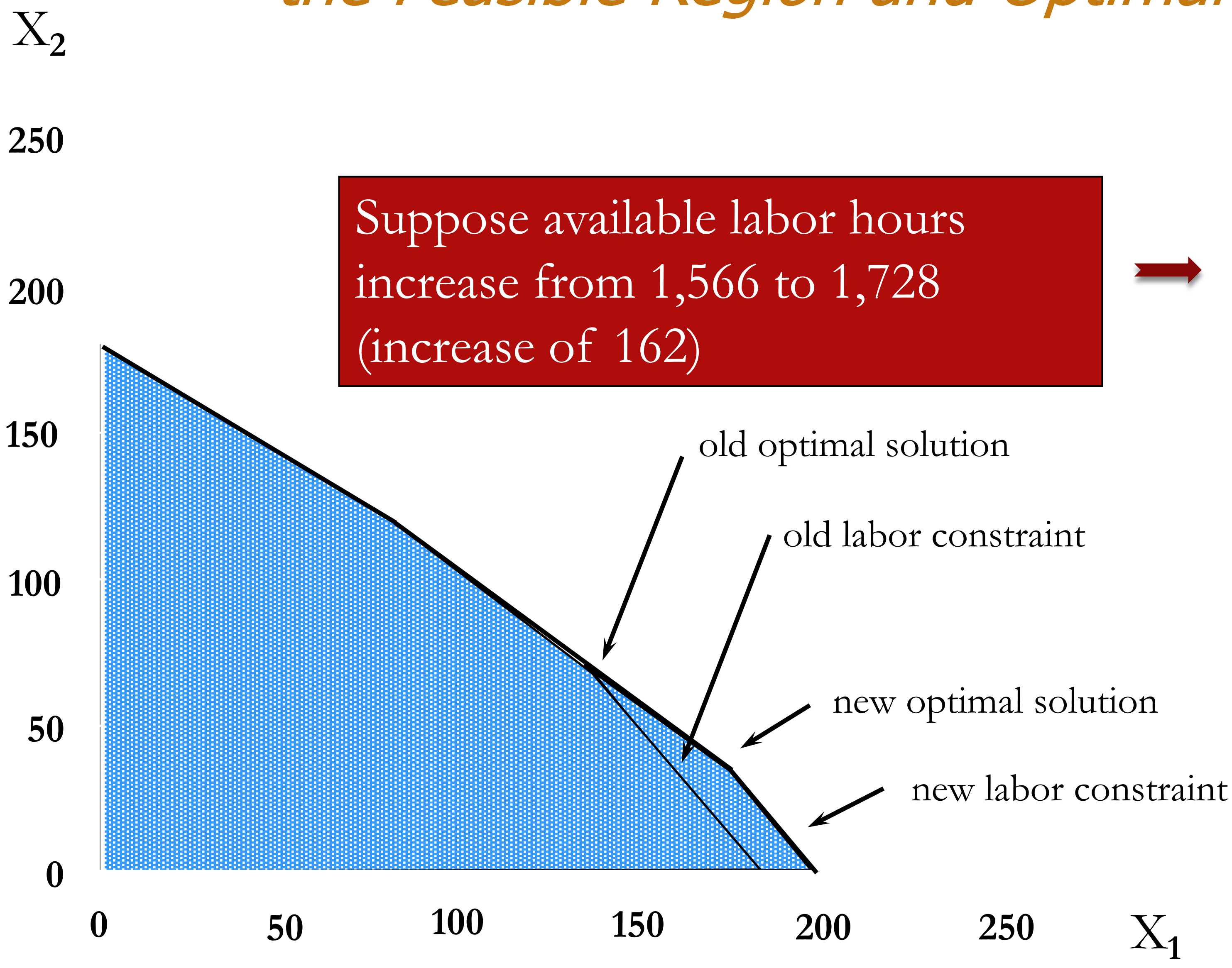
Sensitivity Analysis 2

Changes in Constraint RHS Values

- The *shadow price* of a constraint indicates the amount by which the objective function value changes given a unit *increase* in the RHS value of the constraint, *assuming all other coefficients remain constant*.
- Shadow prices hold only within RHS changes falling within a range
- Shadow prices for nonbinding constraints are always zero.

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Pumps Req'd Used	200	200	200	7	26
Labor Req'd Used	1566	17	1566	234	126
Tubing Req'd Used	2712	0	2880	1E+30	168

How Changing an RHS Value Can Change the Feasible Region and Optimal Solution



Suppose available labor hours increase from 1,566 to 1,728 (increase of 162)

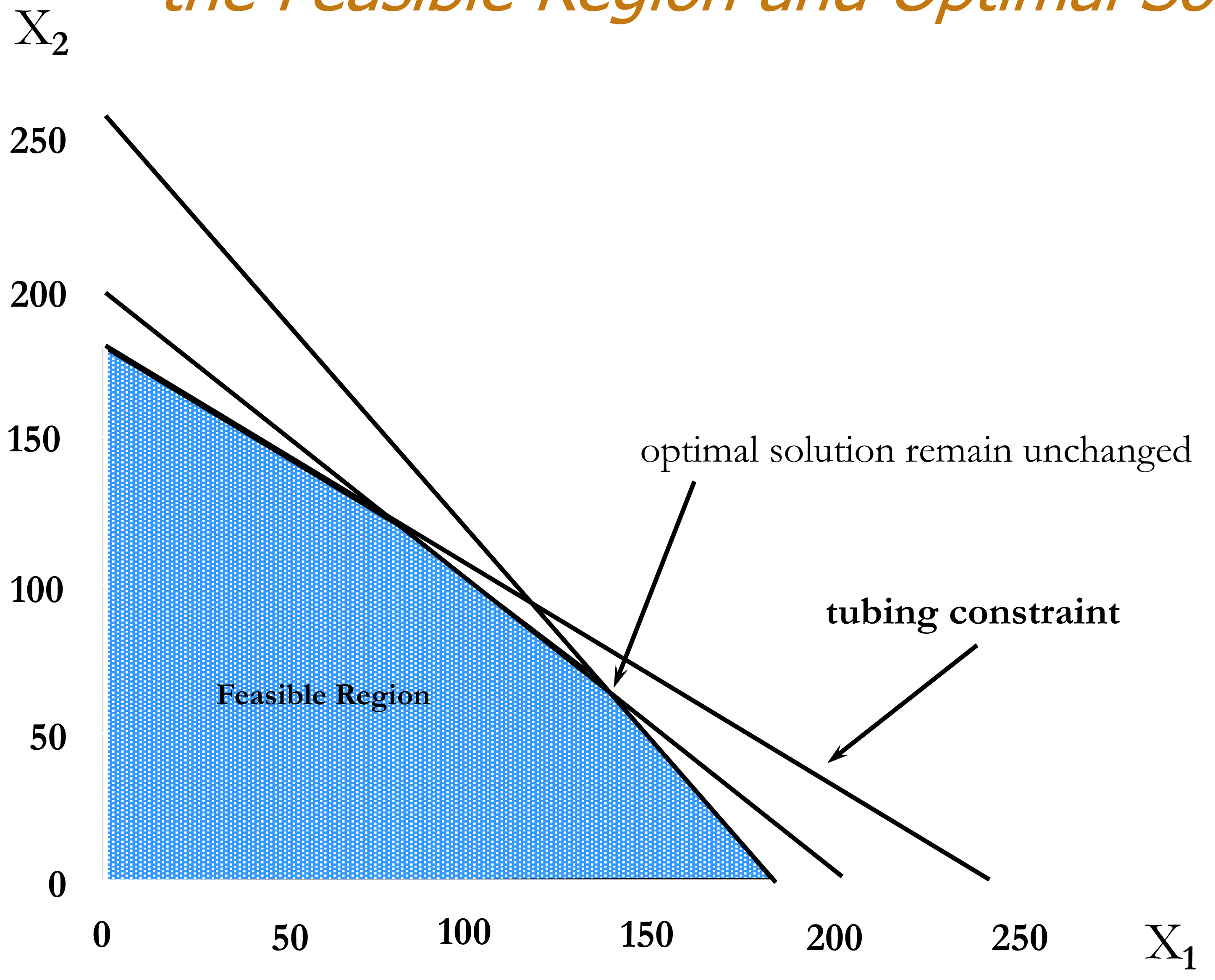
Shadow price = 16,67

New objective function = 66.100+16,67x162 = 68.800

Comments

- Shadow prices only indicate the changes that occur in the objective function value as RHS values change.
- Changing a RHS value for a binding constraint also changes the feasible region and the optimal solution (see following slide).
- To find the optimal solution after changing a binding RHS value, you must re-solve the problem.

How Changing an RHS Value Can Change the Feasible Region and Optimal Solution



Other Uses of Shadow Prices

- Suppose a new Hot Tub (the Typhoon-Lagoon) is being considered. It generates a marginal profit of $320 and requires:
  - 1 pump (shadow price = $200)
  - 8 hours of labor (shadow price = $16.67)
  - 13 feet of tubing (shadow price = $0)
- Q: Would it be profitable to produce any?

A: $320 - $200*1 - $16.67*8 - $0*13 = -$13.33 = ***No!***

The Meaning of Reduced Costs

- The *reduced cost* for each product equals its per-unit marginal profit minus the per-unit value of the resources it consumes (priced at their shadow prices).

Type of Problem	Optimal Value of Decision Variable	Optimal Value of Reduced Cost
Maximization	at simple lower bound	<=0
	between lower & upper bounds	=0
	at simple upper bound	>=0
Minimization	at simple lower bound	>=0
	between lower & upper bounds	=0
	at simple upper bound	<=0

Key Points - I

- The shadow prices of resources equate the marginal value of the resources consumed with the marginal benefit of the goods being produced.

- Resources in excess supply have a shadow price (or marginal value) of zero.

Key Points-II

- The reduced cost of a product is the difference between its marginal profit and the marginal value of the resources it consumes.

- Products whose marginal profits are less than the marginal value of the goods required for their production will not be produced in an optimal solution.

Analyzing Changes in Constraint Coefficients

- Q: Suppose a Typhoon-Lagoon required only 7 labor hours rather than 8. Is it now profitable to produce any?

  A: \$320 - \$200*1 - \$16.67*7 - \$0*13 = \$3.31 = *Yes!*

- Q: What is the maximum amount of labor Typhoon-Lagoons could require and still be profitable?

  A: We need \$320 - \$200*1 - \$16.67*L_3 - \$0*13 >=0

  The above is true if L_3 <= \$120/\$16.67 = \$7.20

Simultaneous Changes in Objective Function Coefficients

- The *100% Rule* can be used to determine if the optimal solutions changes when more than one objective function coefficient changes.

- Two cases can occur:
  - Case 1: All variables with changed obj. coefficients have nonzero reduced costs.
  - Case 2: At least one variable with changed obj. coefficient has a reduced cost of zero.

Simultaneous Changes in Objective Function Coefficients: Case 1

(All variables with changed obj. coefficients have nonzero reduced costs.)

- The current solution remains optimal provided the obj. coefficient changes are all within their allowable range.

Simultaneous Changes in Objective Function Coefficients: Case 2

(At least one variable with changed obj. coefficient has a reduced cost of zero.)

- For each variable compute:

$$r_j = \begin{cases} \dfrac{\Delta c_j}{I_j}, \text{if } \Delta c_j \geq 0 \\[2ex] \dfrac{-\Delta c_j}{D_j}, \text{if } \Delta c_j < 0 \end{cases}$$

- If more than one objective function coefficient changes, the current solution remains optimal provided the r_j sum to <= 1.
- If the r_j sum to > 1, the current solution, might remain optimal, but this is not guaranteed.

A Warning About Degeneracy

- The solution to an LP problem is degenerate if the Allowable Increase of Decrease on any constraint is zero (0).

- When the solution is degenerate:
  1. The methods mentioned earlier for detecting alternate optimal solutions cannot be relied upon.
  2. The reduced costs for the changing cells may not be unique. Also, the objective function coefficients for changing cells must change by at least as much as (and possibly more than) their respective reduced costs before the optimal solution would change.

- When the solution is degenerate (cont’d):

  3. The allowable increases and decreases for the objective function coefficients still hold and, in fact, the coefficients may have to be changed beyond the allowable increase and decrease limits before the optimal solution changes.

  4. The given shadow prices and their ranges may still be interpreted in the usual way but they may not be unique. That is, a different set of shadow prices and ranges may also apply to the problem (even if the optimal solution is unique).