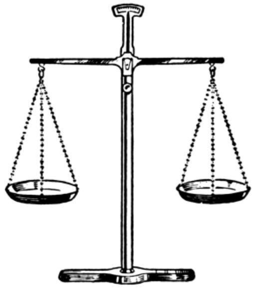


Sensitivity Analysis

Introduction

- When solving an LP problem we assume that values of all model coefficients are known with certainty.
- Such certainty rarely exists.
- Sensitivity analysis helps answer questions about how sensitive the optimal solution is to changes in various coefficients in a model.

General Form of a Linear Programming (LP) Problem

MAX (or MIN): $c_1X_1 + c_2X_2 + \ldots + c_nX_n$

Subject to:

$$a_{11}X_1 + a_{12}X_2 + \ldots + a_{1n}X_n \le b_1$$
$$\vdots$$
$$a_{k1}X_1 + a_{k2}X_2 + \ldots + a_{kn}X_n \le b_k$$
$$\vdots$$
$$a_{m1}X_1 + a_{m2}X_2 + \ldots + a_{mn}X_n = b_m$$

- How sensitive is a solution to changes in the c_i, a_{ij}, and b_i?

Approaches to Sensitivity Analysis

- Change the data and re-solve the model !
  - Sometimes this is the only practical approach.

- Solver (simplex method) also produces sensitivity reports that can answer various questions…

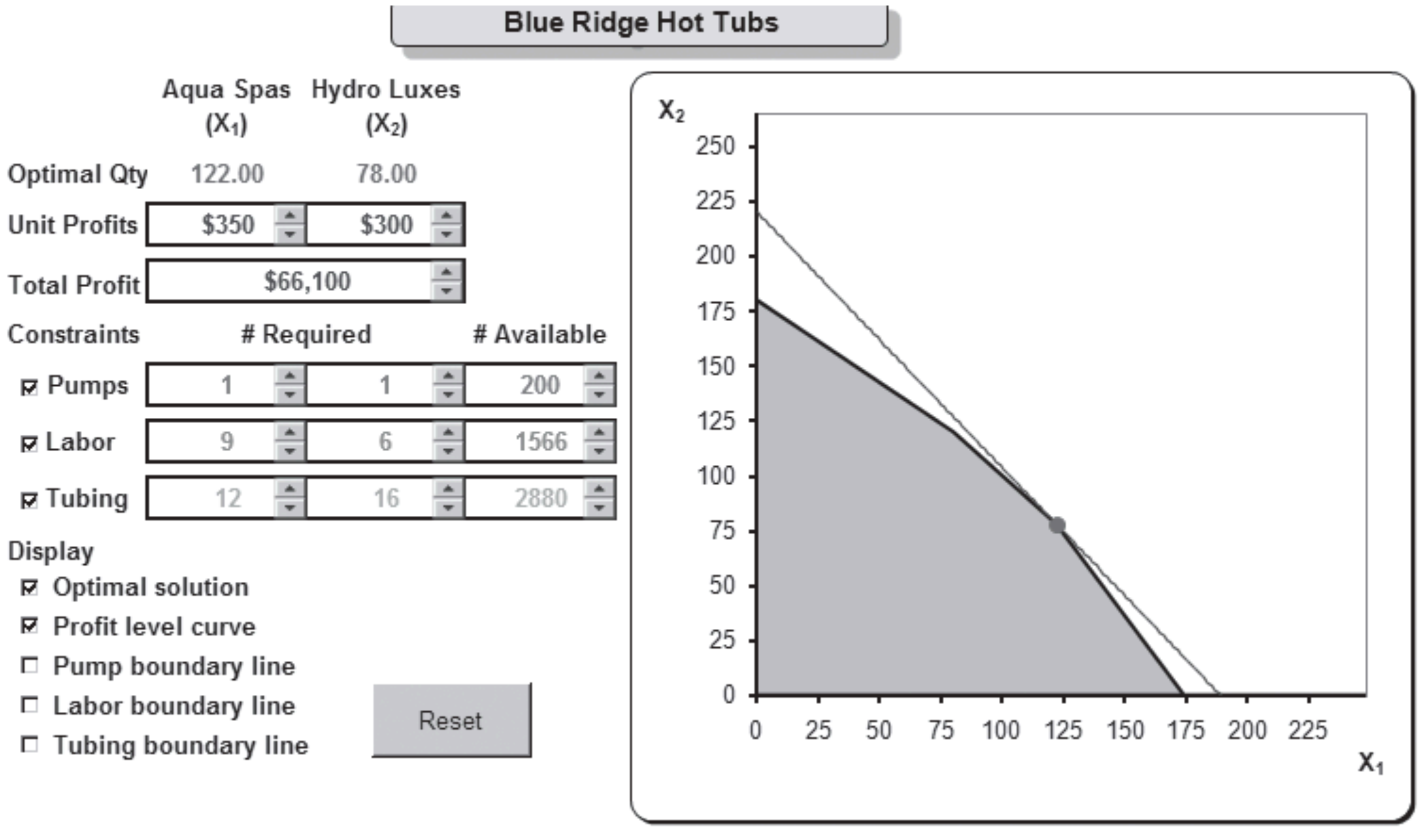
Solver's Sensitivity Report

- Answers questions about:
  - Amounts by which objective function coefficients can change without changing the optimal solution.
  - The impact on the optimal objective function value of changes in constrained resources.
  - The impact on the optimal objective function value of forced changes in decision variables.
  - The impact changes in constraint coefficients will have on the optimal solution.

Once Again, We'll Use The Blue Ridge Hot Tubs Example...

MAX:	$350X_1 + 300X_2$	} profit
S.T.:	$1X_1 + 1X_2 <= 200$	} pumps
	$9X_1 + 6X_2 <= 1566$	} labor
	$12X_1 + 16X_2 <= 2880$	} tubing
	$X_1, X_2 >= 0$	} nonnegativity

Slack variables



Slack variables

MAX:	$350X_1 + 300X_2$	} profit
S.T.:	$1X_1 + 1X_2 + S_1 = 200$	} pumps
	$9X_1 + 6X_2 + S_2 = 1566$	} labor
	$12X_1 + 16X_2 + S_3 = 2880$	} tubing
	$X_1, X_2, S_1, S_2, S_3 >= 0$	} nonnegativity

If the slack variables of the constraint is 0 then the constraint is binding

Binding constraints prevent the objective funcion to achieve higher values

The values of the slack variables indicate the difference between the LHS and RHS of each constraint

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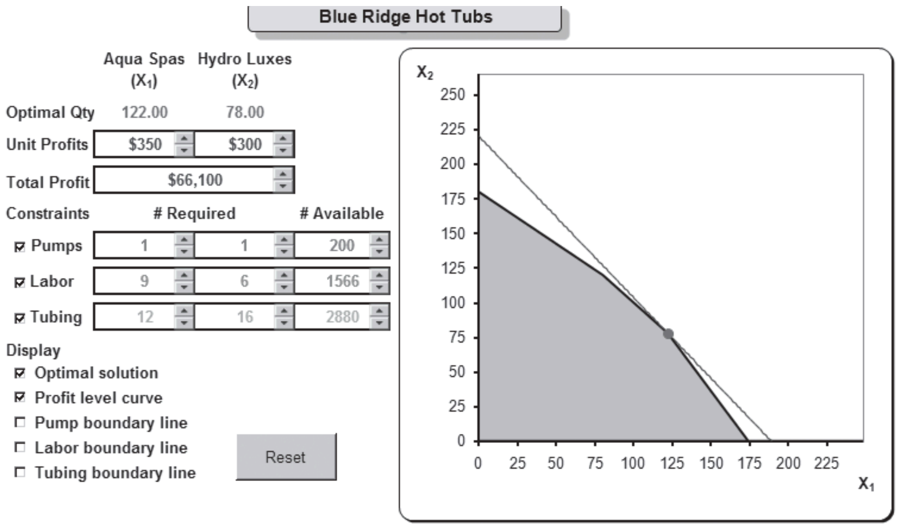
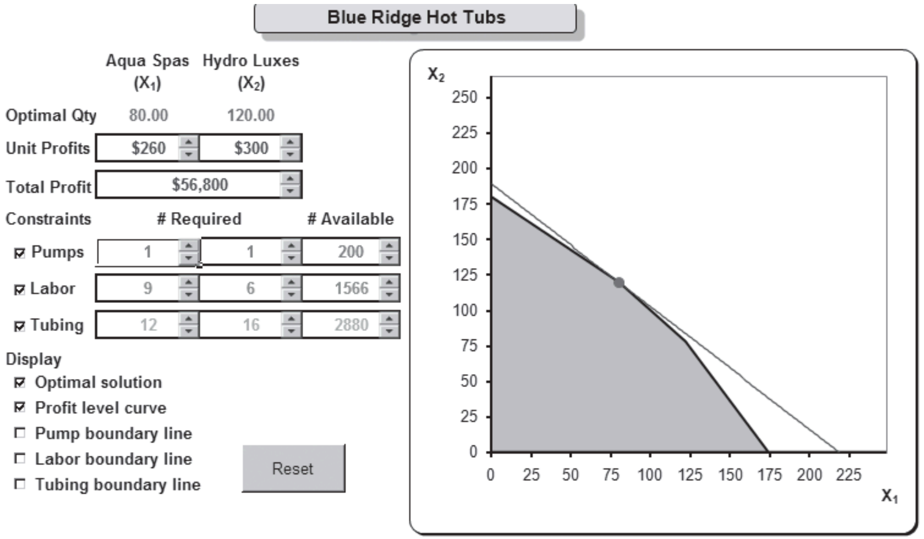
How Changes in Objective Coefficients Change the Slope of the Level Curve





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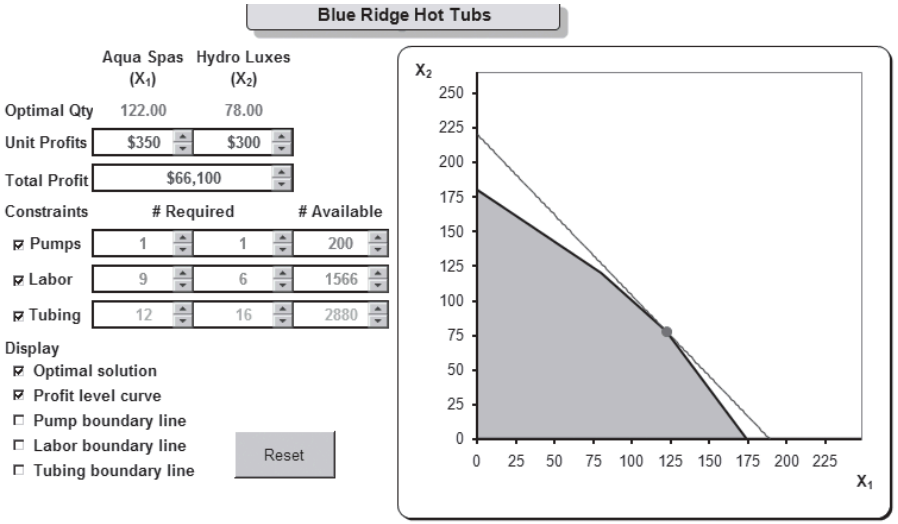

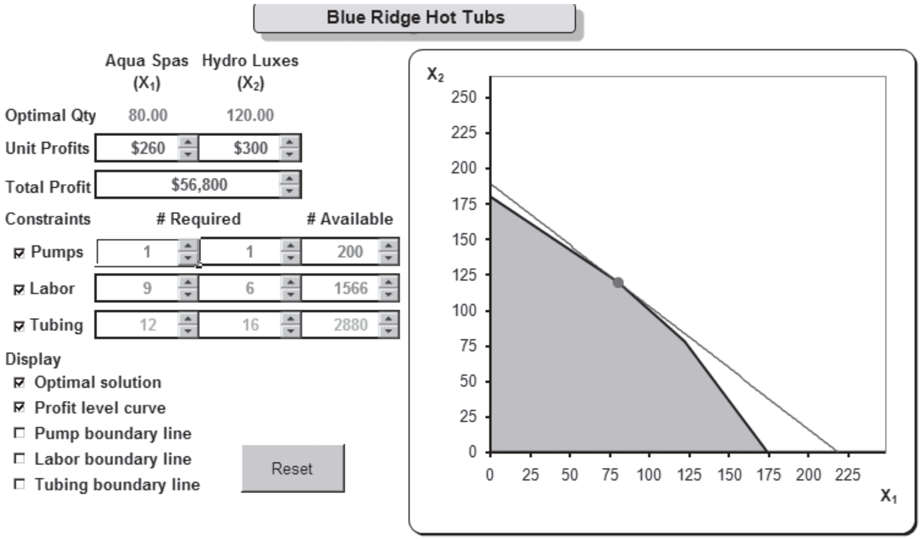

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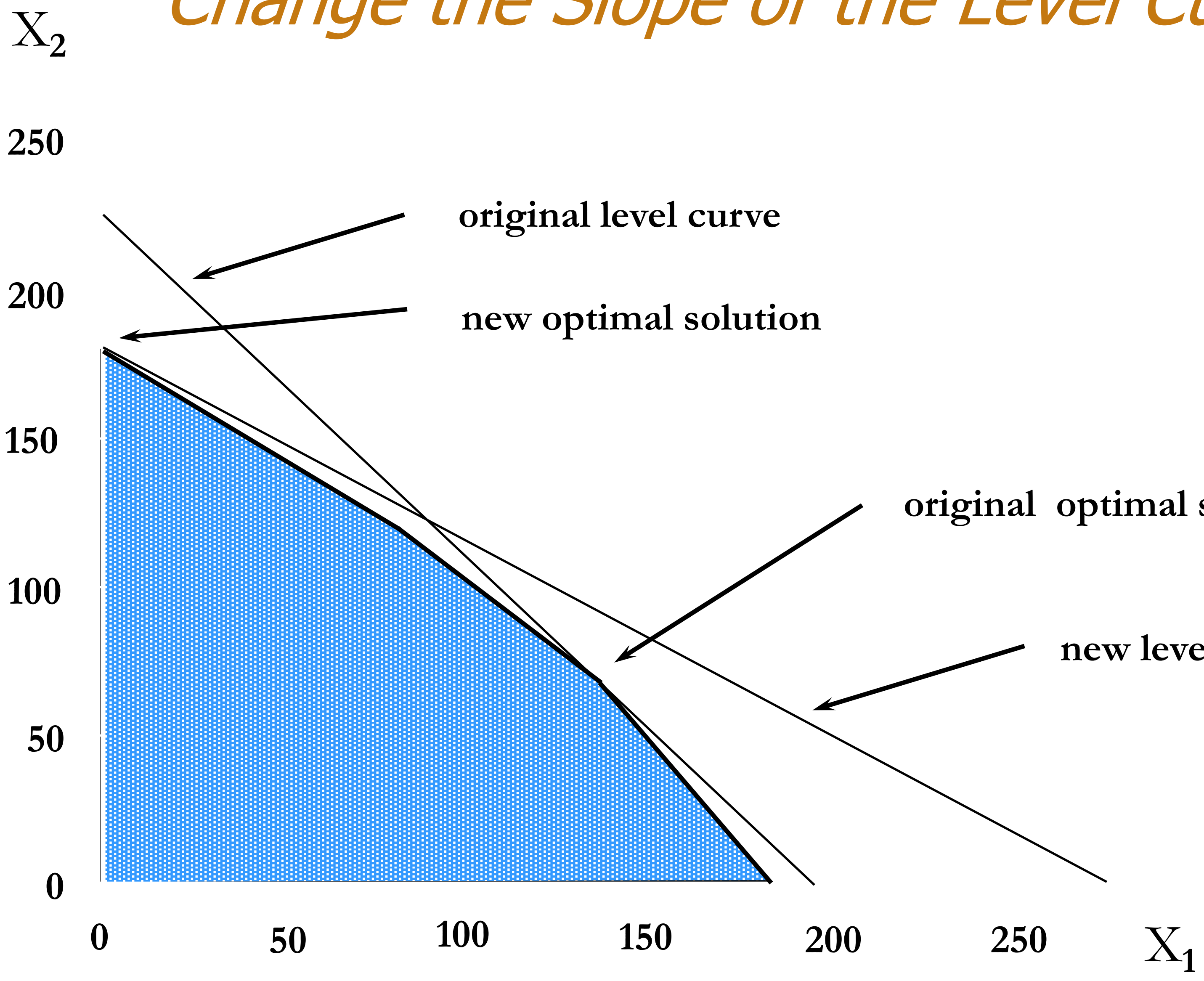

Sensitivity Report

```
printSensitivityObj(model)
## Objs Sensitivity
## 1 C1 300 <= C1 <= 450
## 2 C2 233.333333333333 <= C2 <= 350
```

The objective coefficient C1 was 350 therefore it can increase up to $100 or decrease up to $50 without changing the optimal solution assuming all other coefficients remain constant

Similarly, the objective function value C2 can increase by $50 or decrease by $66.67 without changing the optimal values of the decision variables, assuming all other coefficients remain constant.

Sensitivity Report

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Sensitivity Report

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Number to make Aqua-Spas	122	0	350	100	50
Number to make Hydro-Luxes	78	0	300	50	66.66667

The objective coefficient C1 was 350 therefore it can increase up to $100 or decrease up to $50 without changing the optimal solution assuming all other coefficients remain constant

Similarly, the objective function value C2 can increase by $50 or decrease by $66.67 without changing the optimal values of the decision variables, assuming all other coefficients remain constant.

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Sensitivity Report

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```

These are called optimality ranges indicate the amounts by which an objective function coefficient can change without changing the optimal solution, *assuming all other coefficients remain constant.*

When objective function coefficient are changed the feasible region remain unchanged.

Alternate Optimal Solutions

If the range of this variability (increase or decrease) is 0 then an alternate optimal solution exists (if we are not in the special case of a degenerate solution)

That is, it means that, also a little change of the coefficient may cause a change of the optimal solution