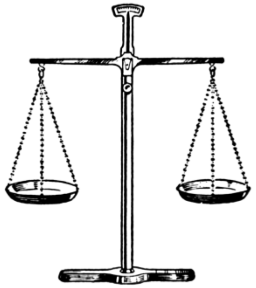

The simplex method

The Simplex Method

- To use the simplex method, we first convert all inequalities to equalities by adding slack variables to <= constraints and subtracting slack variables from >= constraints.

For example: $a_{k1}X_1 + a_{k2}X_2 + \ldots + a_{kn}X_n <= b_k$

converts to: $a_{k1}X_1 + a_{k2}X_2 + \ldots + a_{kn}X_n + S_k = b_k$

And: $a_{k1}X_1 + a_{k2}X_2 + \ldots + a_{kn}X_n >= b_k$

converts to: $a_{k1}X_1 + a_{k2}X_2 + \ldots + a_{kn}X_n - S_k = b_k$

For Our Example Problem...

$$\text{MAX: } 350X_1 + 300X_2 \quad \} \text{ profit}$$

$$\text{S.T.: } 1X_1 + 1X_2 + S_1 = 200 \quad \} \text{ pumps}$$

$$9X_1 + 6X_2 + S_2 = 1566 \quad \} \text{ labor}$$

$$12X_1 + 16X_2 + S_3 = 2880 \quad \} \text{ tubing}$$

$$X_1, X_2, S_1, S_2, S_3 >= 0 \quad \} \text{ nonnegativity}$$

- If there are n variables in a system of m equations (where $n \geq m$) we can select any m variables and solve the equations (setting the remaining n-m variables to zero.)

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Possible Basic Feasible Solutions

	Basic Variables	Nonbasic Variables	Solution	Objective Value
1	S_1, S_2, S_3	X_1, X_2	X_1=0, X_2=0, S_1=200, S_2=1566, S_3=2880	0
2	X_1, S_1, S_3	X_2, S_2	X_1=174, X_2=0, S_1=26, S_2=0, S_3=792	60,900
3	X_1, X_2, S_3	S_1, S_2	X_1=122, X_2=78, S_1=0, S_2=0, S_3=168	66,100
4	X_1, X_2, S_2	S_1, S_3	X_1=80, X_2=120, S_1=0, S_2=126, S_3=0	64,000
5	X_2, S_1, S_2	X_1, S_3	X_1=0, X_2=180, S_1=20, S_2=486, S_3=0	54,000
6*	X_1, X_2, S_1	S_2, S_3	X_1=108, X_2=99, S_1=-7, S_2=0, S_3=0	67,500
7*	X_1, S_1, S_2	X_2, S_3	X_1=240, X_2=0, S_1=-40, S_2=-594, S_3=0	84,000
8*	X_1, S_2, S_3	X_2, S_1	X_1=200, X2=0, S_1=0, S_2=-234, S_3=480	70,000
9*	X_2, S_2, S_3	X_1, S_1	X_1=0, X_2=200, S_1=0, S_2=366, S_3=-320	60,000
10*	X_2, S_1, S_3	X_1, S2	X_1=0, X_2=261, S_1=-61, S_2=0, S_3=-1296	78,300

* denotes infeasible solutions

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	Basic Variables	Nonbasic Variables	Solution	Objective Value
1	S_1, S_2, S_3	X_1, X_2	X_1=0, X_2=0, S_1=200, S_2=1566, S_3=2880	0
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5	X_2, S_1, S_2	X_1, S_3	X_1=0, X_2=180, S_1=20, S_2=486, S_3=0	54,000
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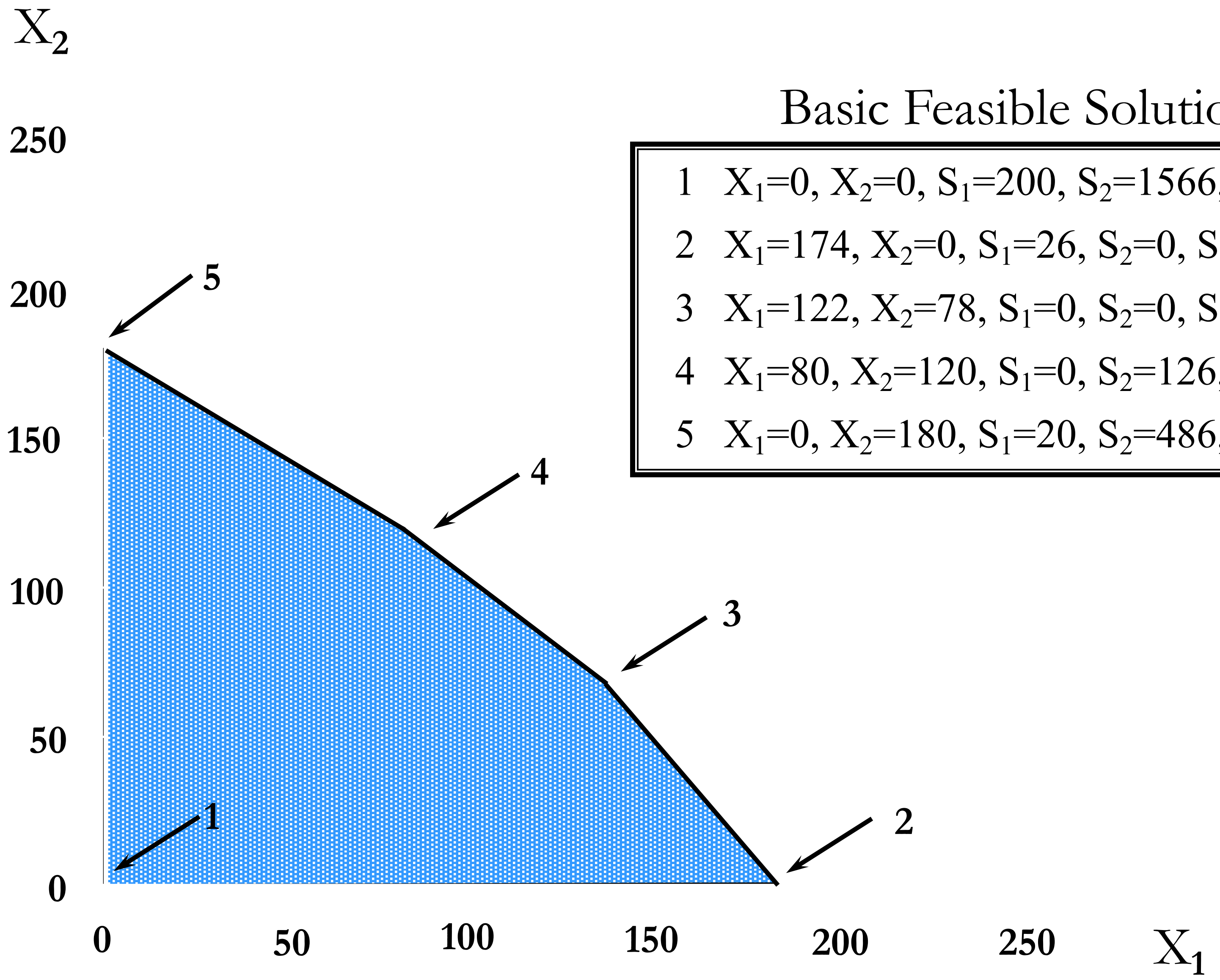
Basic Feasible Solutions & Extreme Points



Basic Feasible Solutions

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2	X_1=174, X_2=0, S_1=26, S_2=0, S_3=792
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4	X_1=80, X_2=120, S_1=0, S_2=126, S_3=0
5	X_1=0, X_2=180, S_1=20, S_2=486, S_3=0

Simplex Method Summary

- Identify any basic feasible solution (or extreme point) for an LP problem, then moving to an adjacent extreme point, if such a move improves the value of the objective function.
- Moving from one extreme point to an adjacent one occurs by switching one of the basic variables with one of the nonbasic variables to create a new basic feasible solution (for an adjacent extreme point).
- When no adjacent extreme point has a better objective function value, stop -- the current extreme point is optimal.