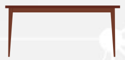
Furniture Factory Problem

Graphical interpretation and solution of an LP problem

GUROBI OPTIMIZATION

LP formulation of furniture problem

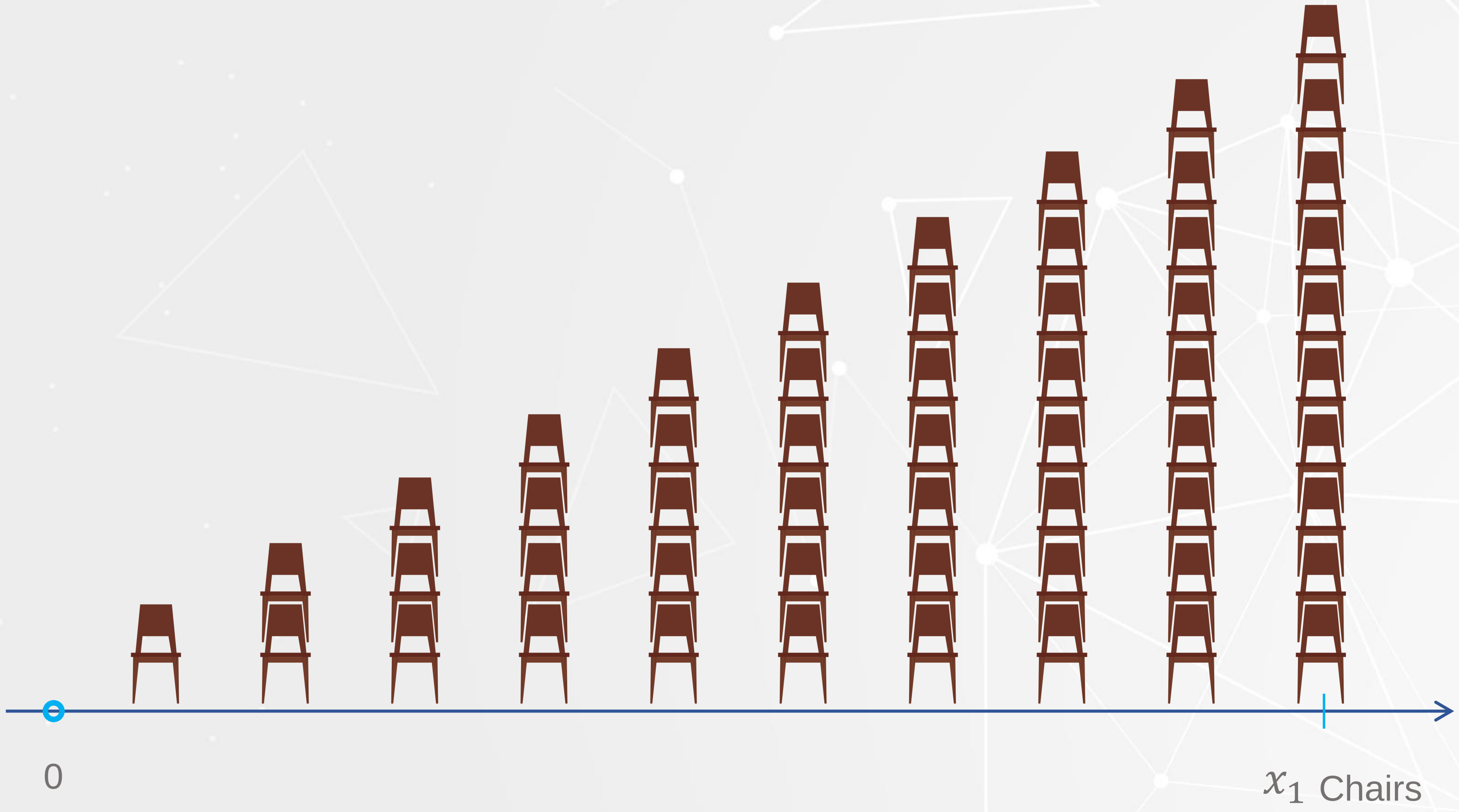
(1.0). Max revenue $= 45x_1 + 80x_2$

(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

Graphical solution of Furniture Problem … 1



(1.0). $\text{Max revenue} = 45x_1 + 80x_2$

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$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

Graphical solution of Furniture Problem … 2

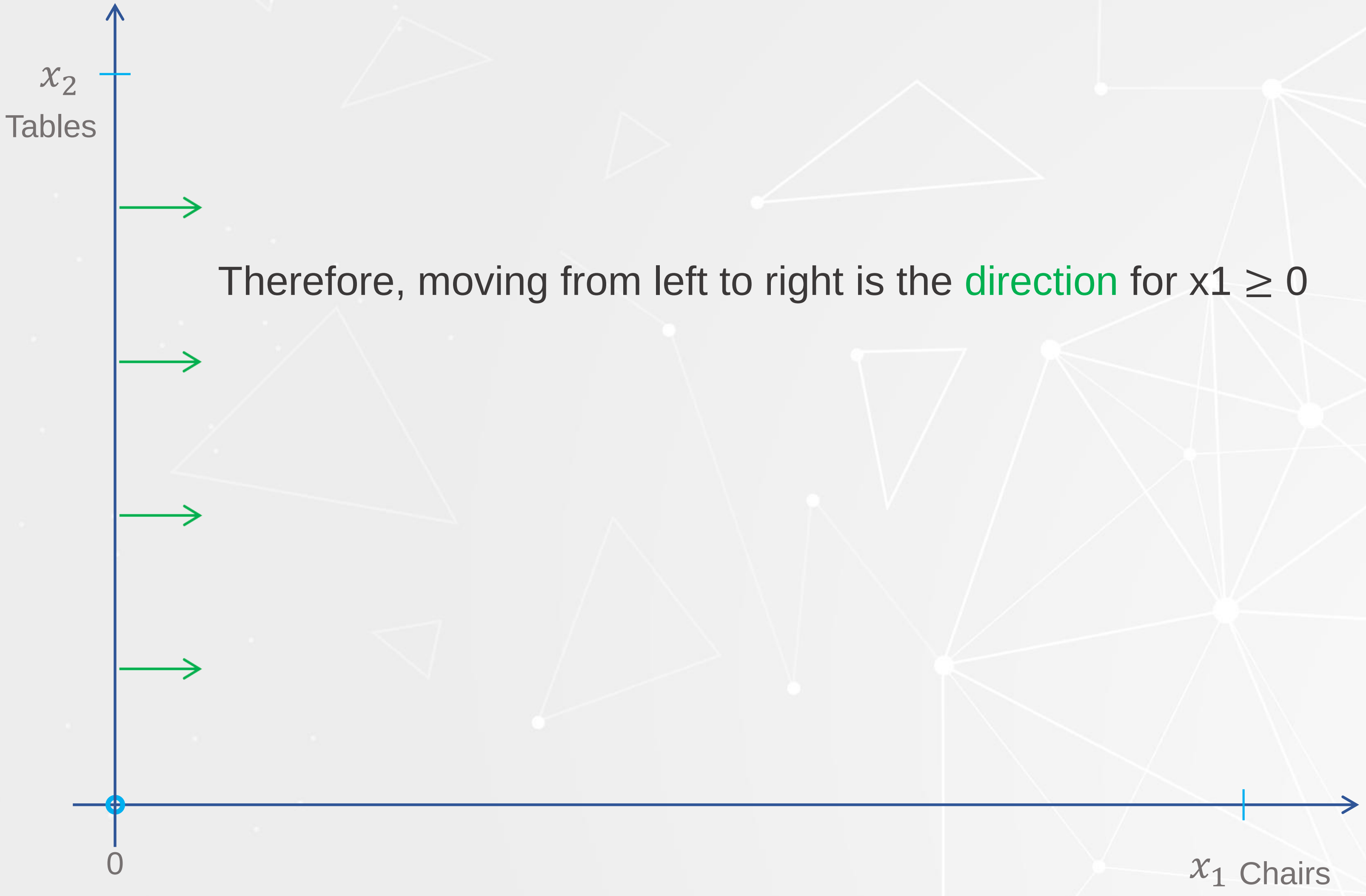
(1.0). Max revenue $= 45x_1 + 80x_2$

(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

Graphical solution of Furniture Problem … 3



Therefore, moving from left to right is the direction for x1 ≥ 0

(1.0). Max revenue $= 45x_1 + 80x_2$

(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

Graphical solution of Furniture Problem … 4

Therefore, moving from low to high is the direction for x2 ≥ 0

(1.0). Max revenue $= 45x_1 + 80x_2$

(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

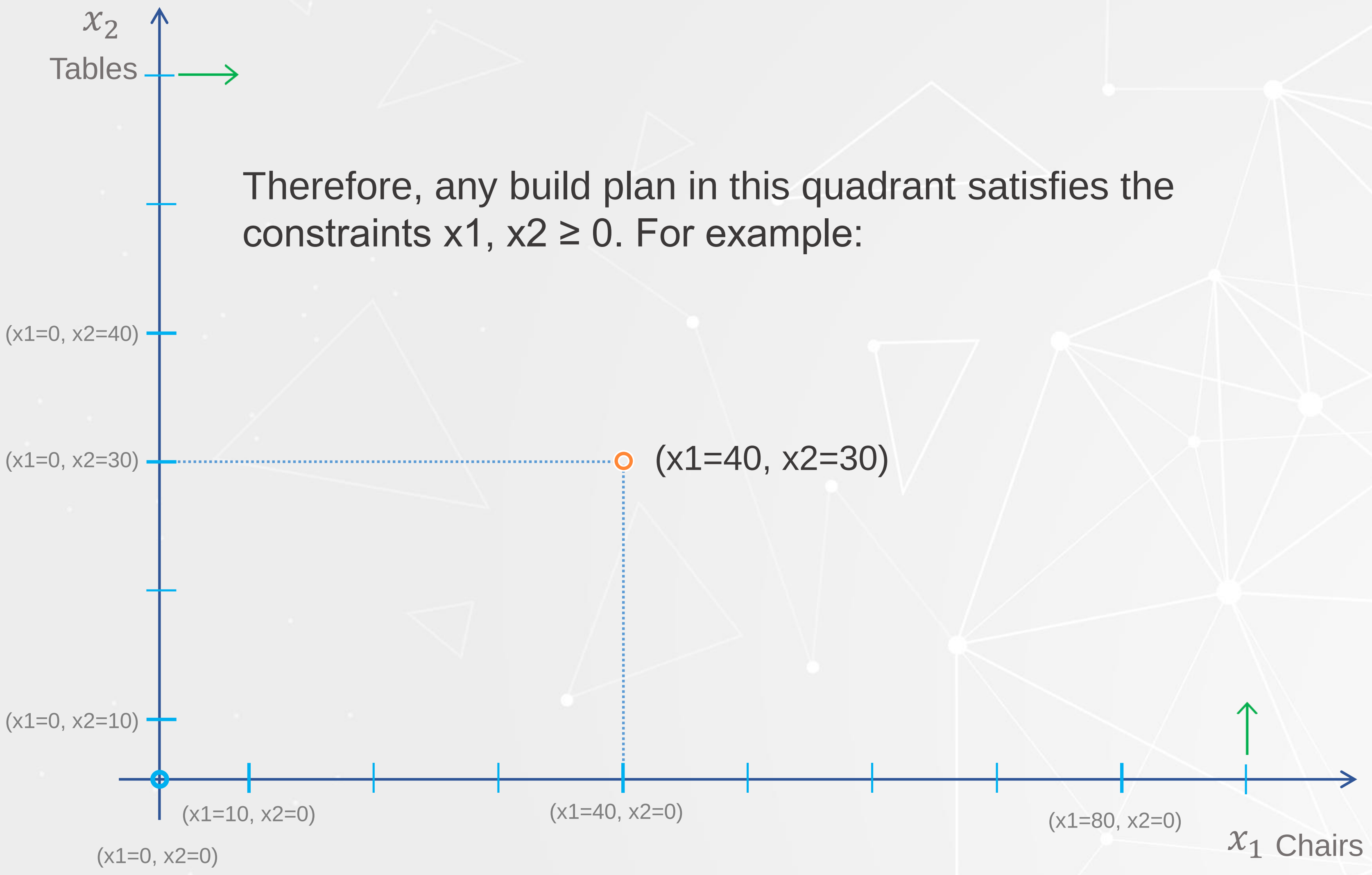
(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

GUROBI OPTIMIZATION

Graphical solution of Furniture Problem … 5

Therefore, any build plan in this quadrant satisfies the constraints x1, x2 ≥ 0. For example:



(1.0). $\text{Max revenue} = 45x_1 + 80x_2$

(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

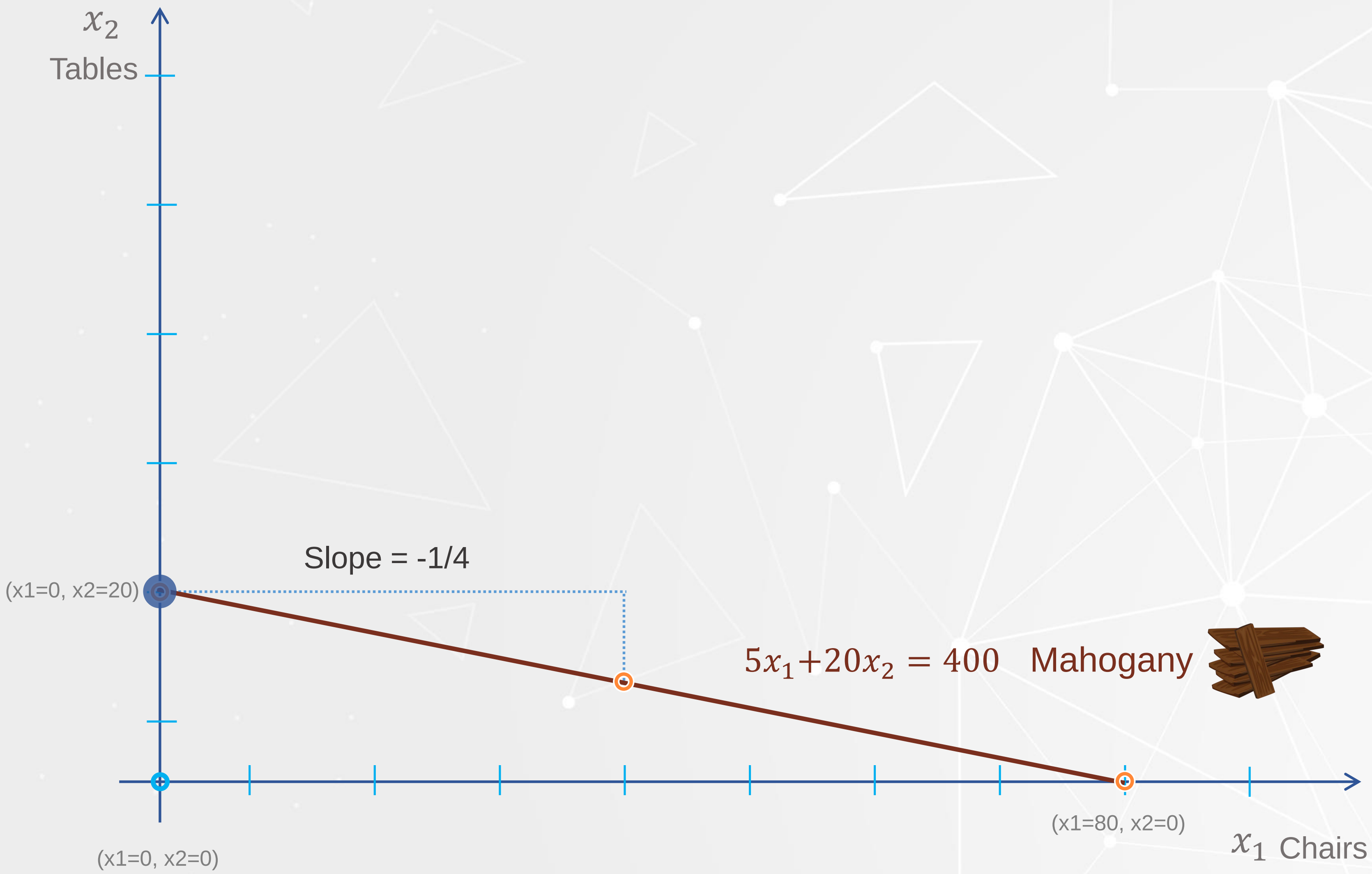
Graphical solution of Furniture Problem … 6

- $5x1 + 20x2 \leq 400$ (mahogany constraint)
- $5x1 + 20x2 = 400$ (mahogany equation)

- Expressing x2 in terms of x1
  - $20x2 = 400 – 5x1$
  - $x2 = 400/20 –(5/20)x1$
  - Hence, $x2 = 20 –(1/4)x1$

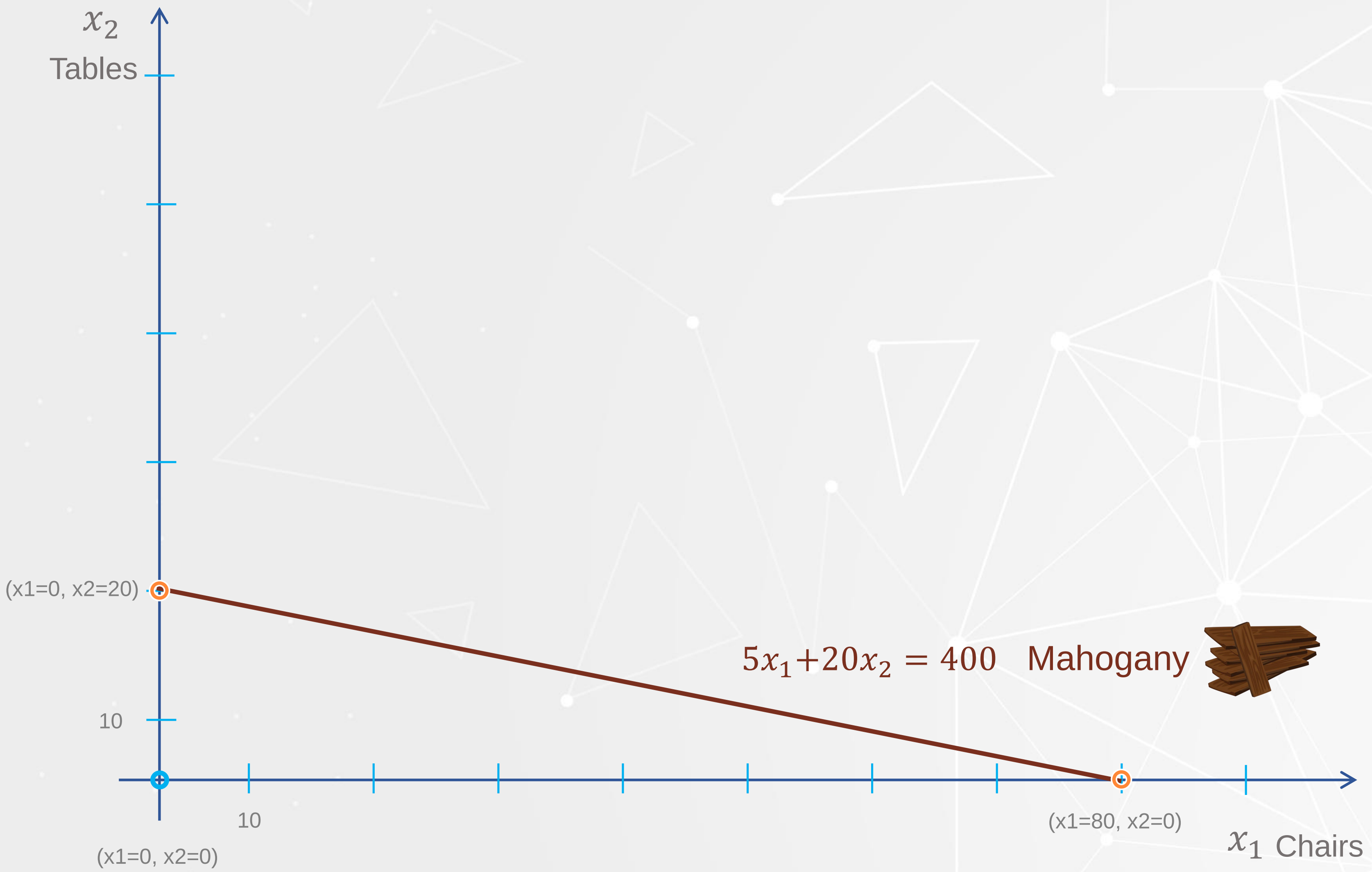
- If (x1 = 0) then (x2 = 20)
- If (x1 = 1) chairs, then x2 = 20 –(1/4)(x1 = 1) = 19.75 tables
- Mahogany tradeoff tables for chairs is (1/4 = 0.25)

Graphical solution of Furniture Problem … 7



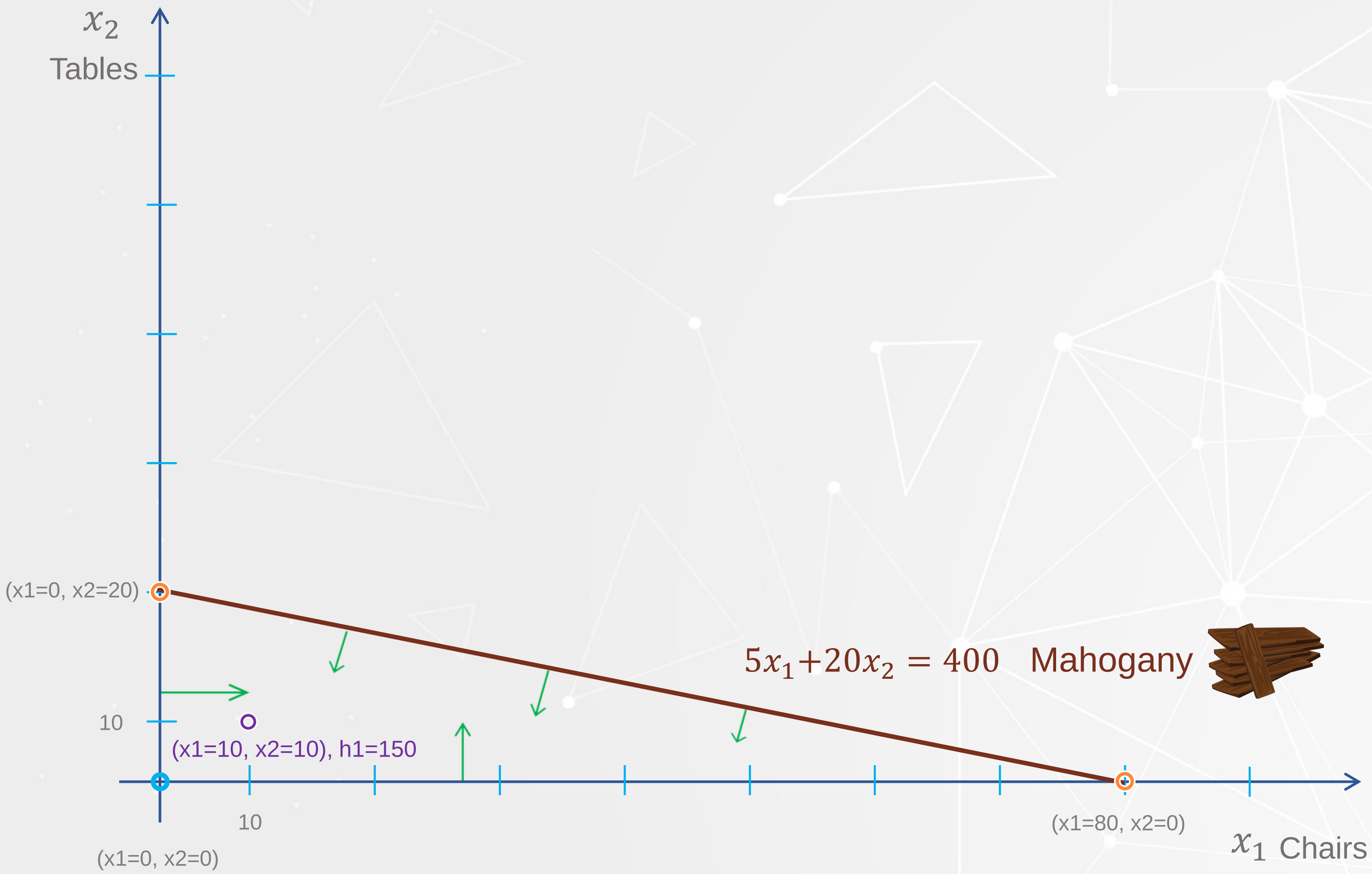
- Let's graph the equation x2 = 20 –(1/4)x1

Graphical solution of Furniture Problem … 8



- Mahogany constraint: $5x1 + 20x2 \leq 400$.
- (slack variable) h1 ≥ 0: amount of unused mahogany for Production Plan (x1, x2)
- Equation representing mahogany constraint $5x1 + 20x2 + h1 = 400$

Graphical solution of Furniture Problem … 8

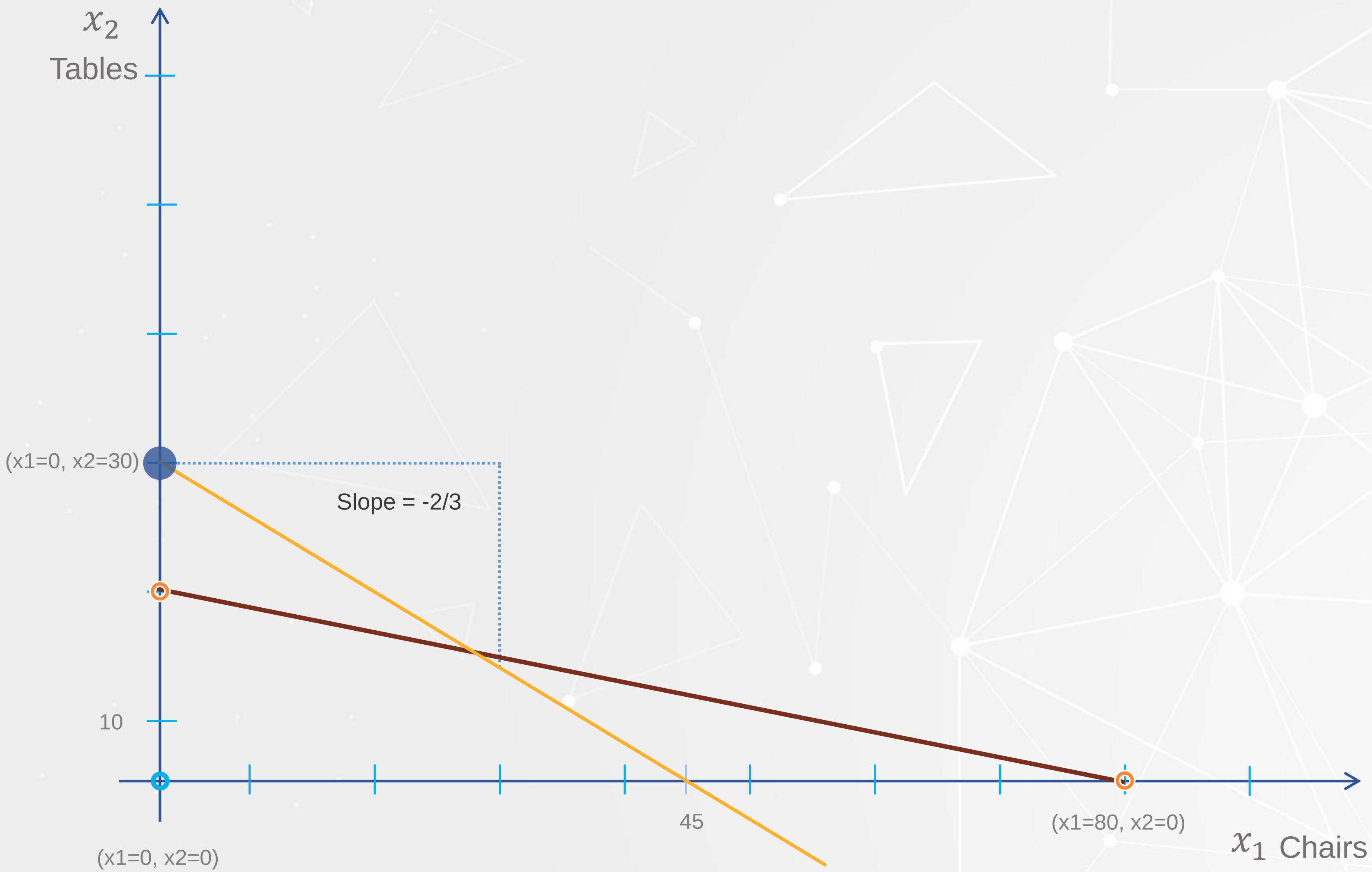


- Consider Production Plan (x1=10, x2=10)
- Value of slack variable h1 = 400 – 5(x1=10) – 20(x2=10)=150

Graphical solution of Furniture Problem … 9

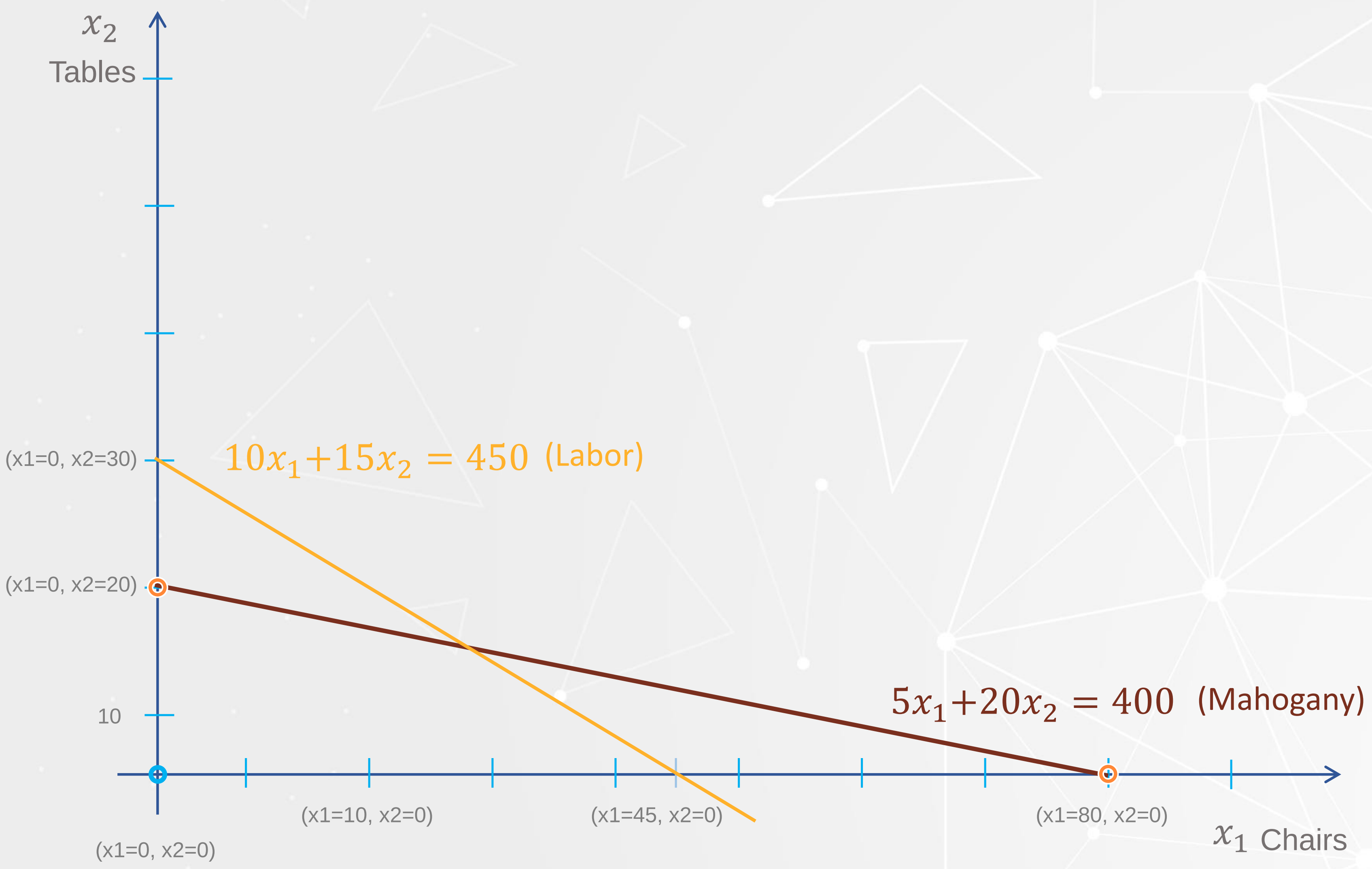
- $10x1 + 15x2 \leq 450$ (labor constraint)
- $10x1 + 15x2 = 450$ (labor equation)
- Expressing x2 in terms of x1
  - $x2 = 30 - (2/3)x1$
- If $(x1 = 0)$ then $(x2 = 30)$
- If $(x1 = 1)$ chair, then $x2 = 30 - (2/3)(x1 = 1) = 29.333$ tables
- Labor tradeoff tables for chairs is $(2/3 = 0.667)$

Graphical solution of Furniture Problem … 10



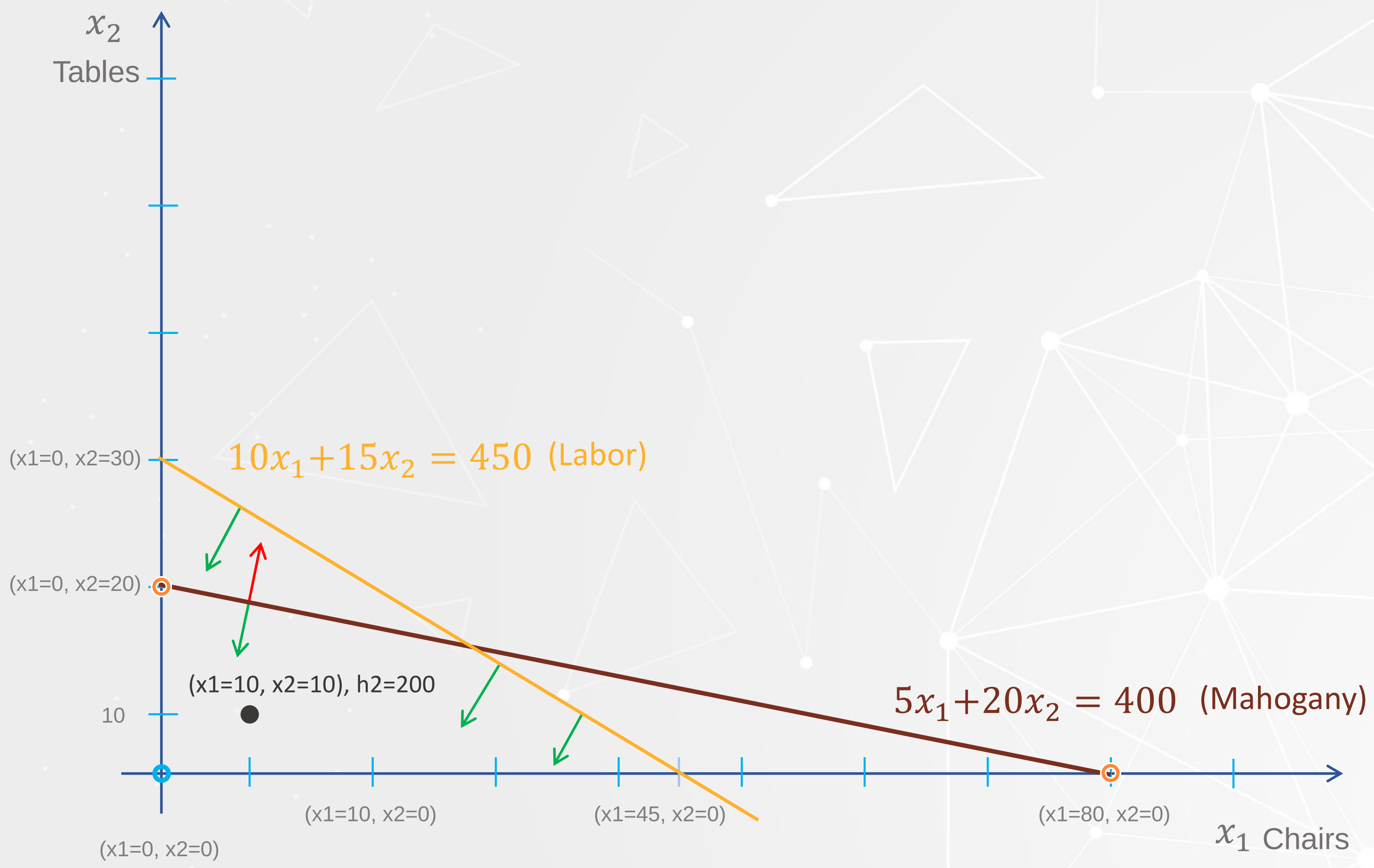
- Let's graph the equation $x2 = 30 - (2/3)x1$

Graphical solution of Furniture Problem … 11



- Labor constraint
  $10x1 + 15x2 \leq 450$
- (slack variable) h2 ≥ 0: amount of unused labor for production plan (x1, x2)
- Equation representing labor constraint
  $10x1 + 15x2 + h2 = 450$

Graphical solution of Furniture Problem … 11



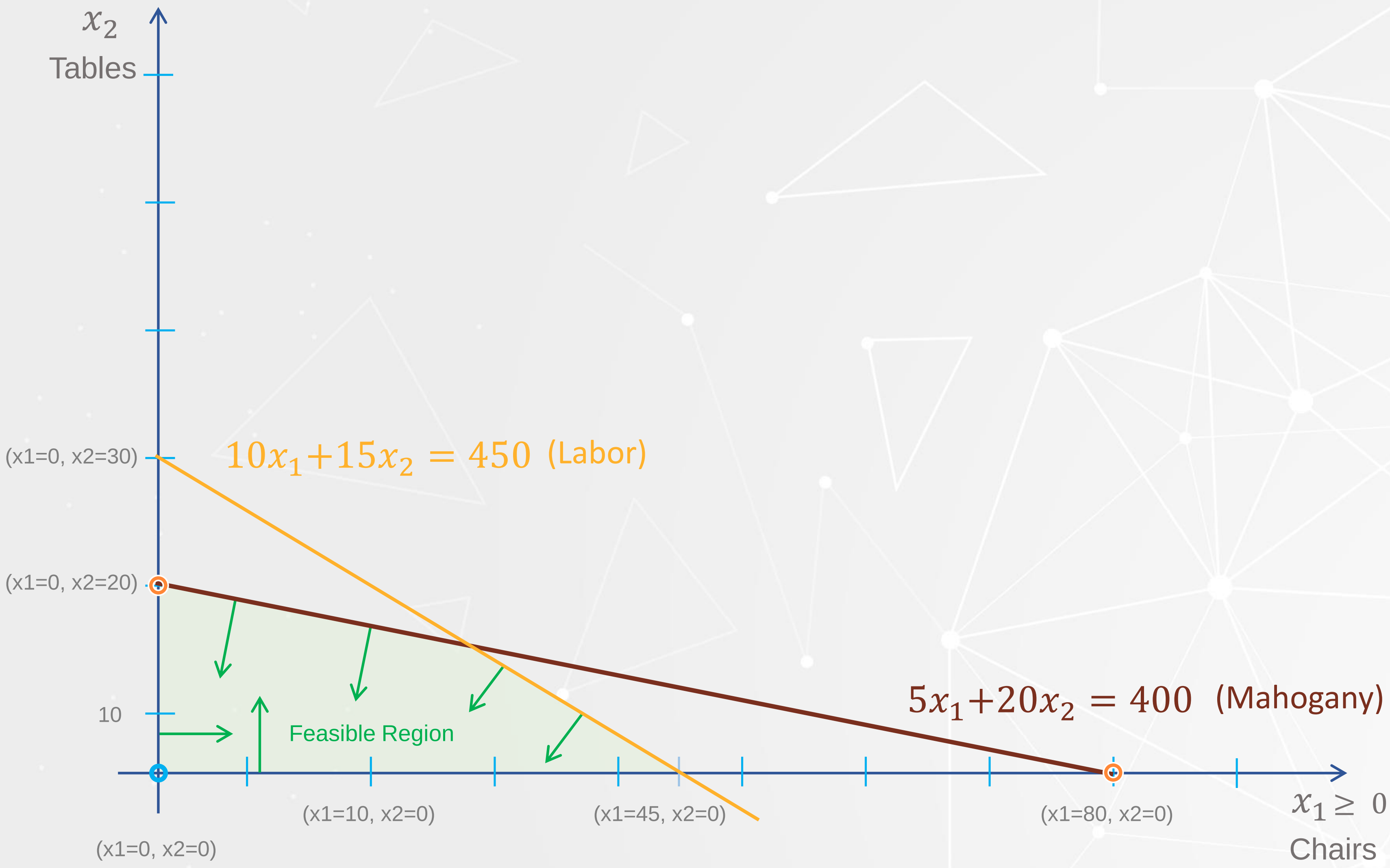
- Production Plan (x1=10, x2=10)
- Slack variable value h2 = 450 – 10(x1=10) – 15(x2=10) = 200

Graphical solution of Furniture Problem … 12

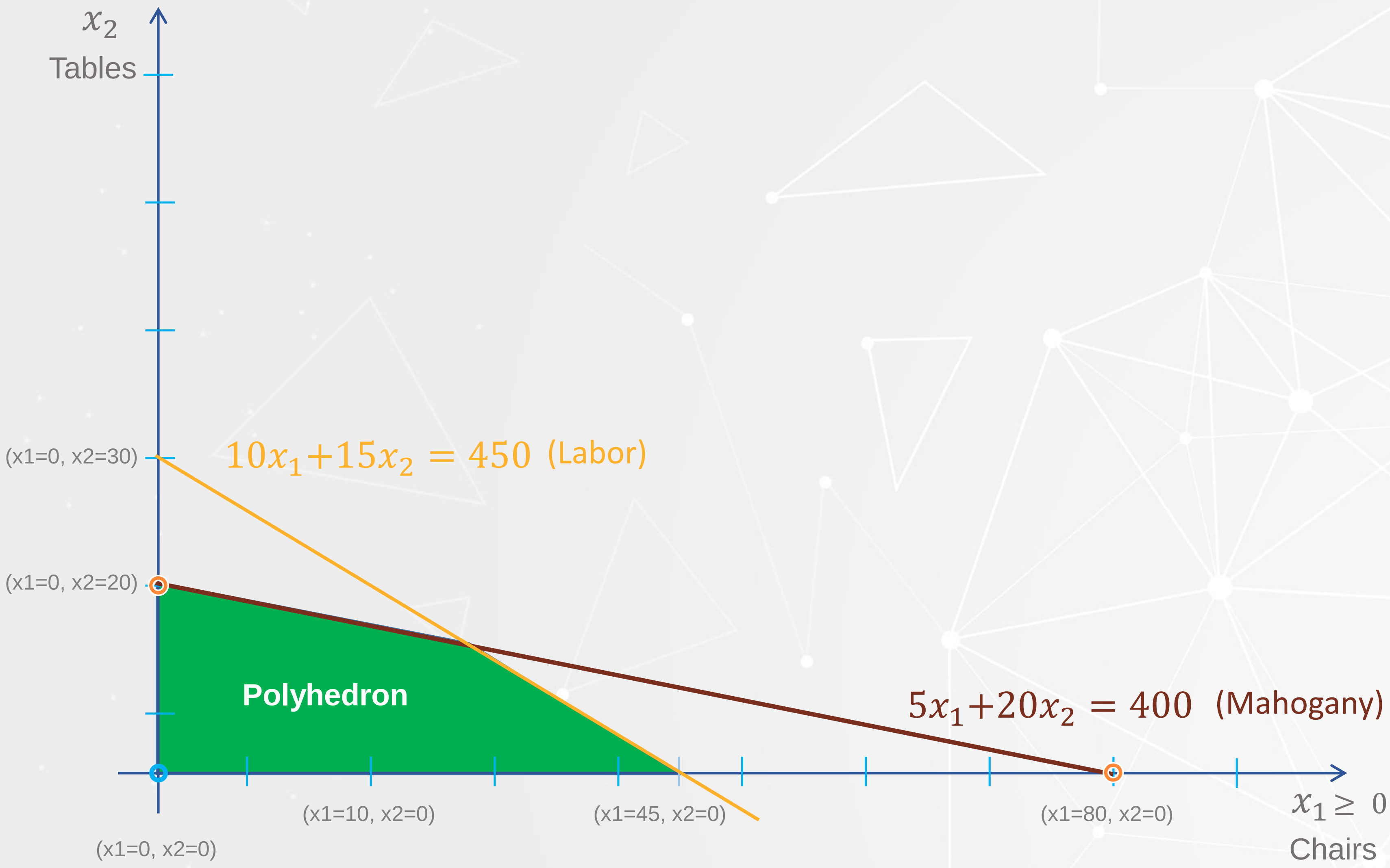
(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$x_1, x_2 \geq 0$ Non – negativity



Graphical solution of Furniture Problem … 13



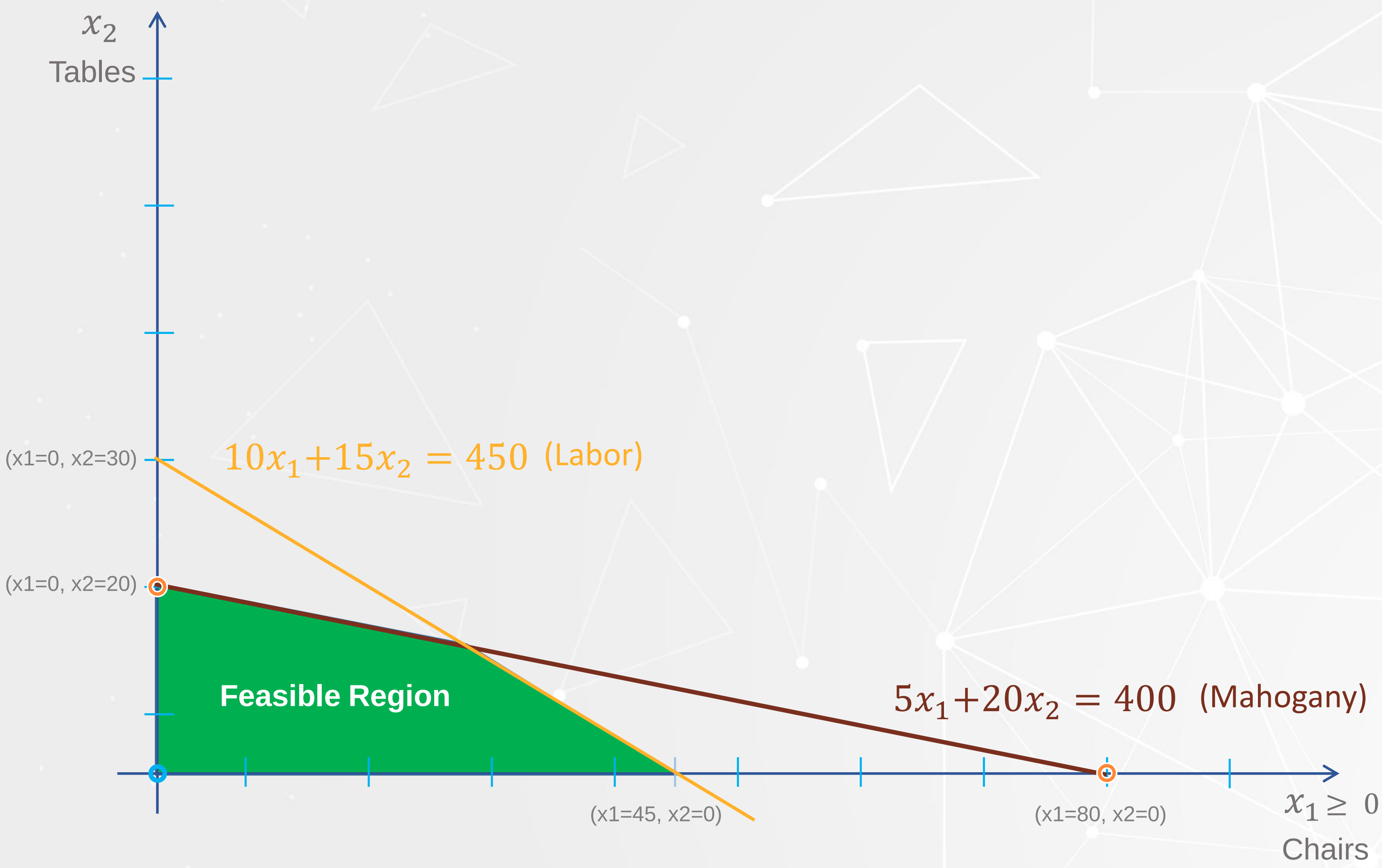
(2.0) $5x_1 + 20x_2 \leq 400$ Units of mahogany capacity

(3.0). $10x_1 + 15x_2 \leq 450$ Labor hours capacity

$\boldsymbol{x_1, x_2 \geq 0}$ Non – negativity

- In the theory of linear programming the feasible region is called a polyhedron.

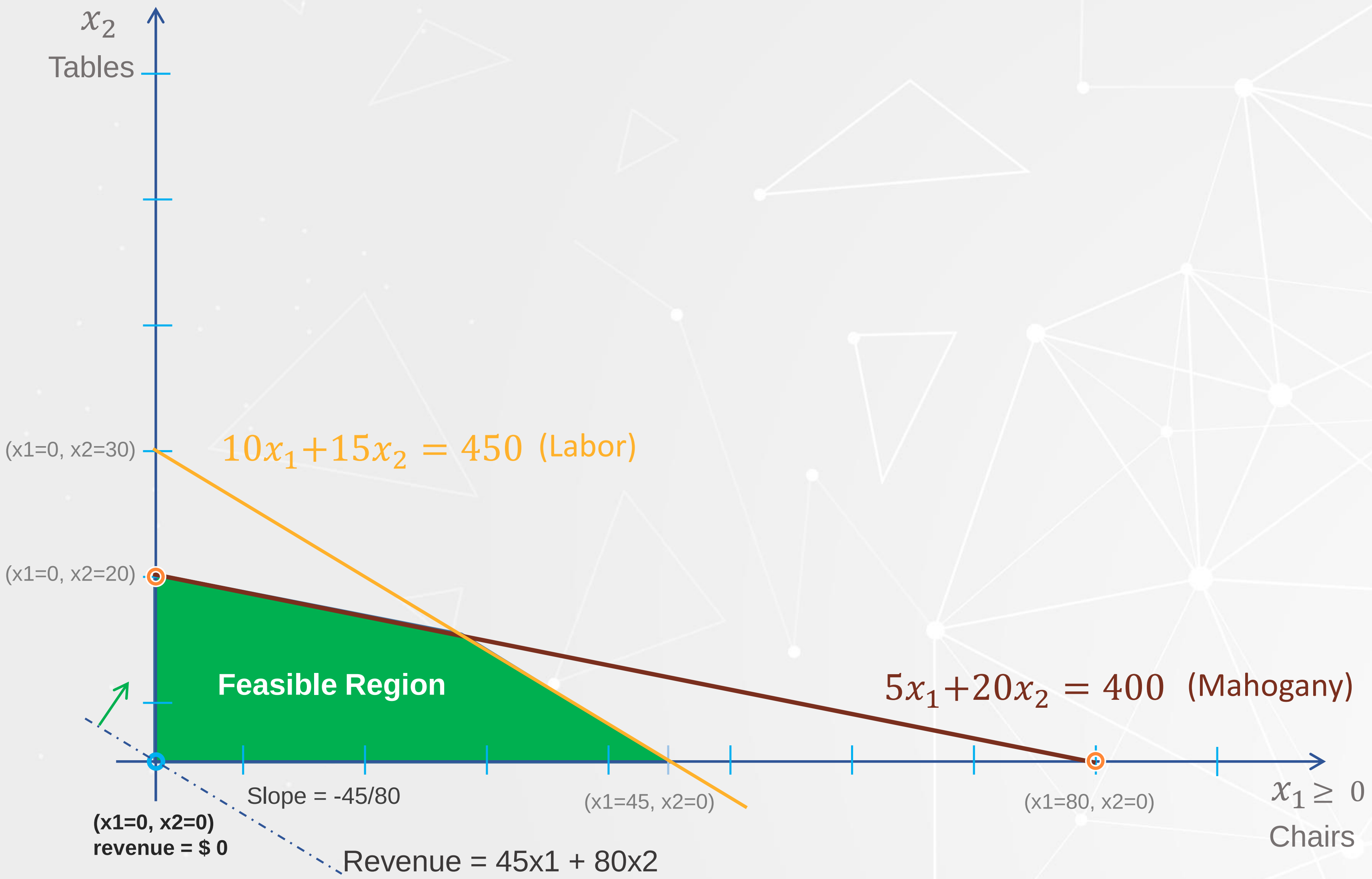
Graphical solution of Furniture Problem … 14



- The objective function: revenue = 45x1 + 80x2

  x2 = revenue/80 –(45/80)x1

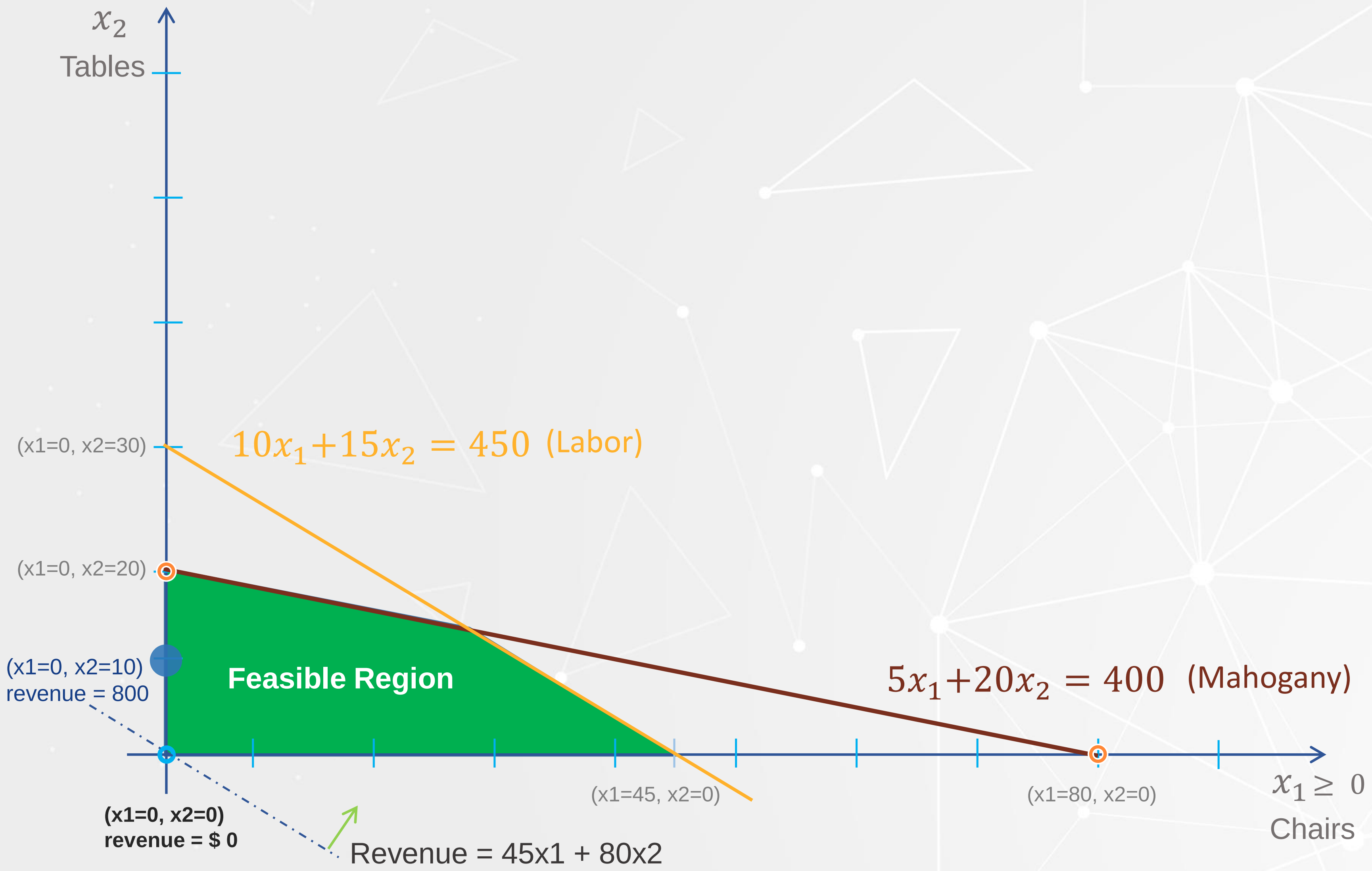
Graphical solution of Furniture Problem … 14



x2 =
revenue/80 –(45/80)x1

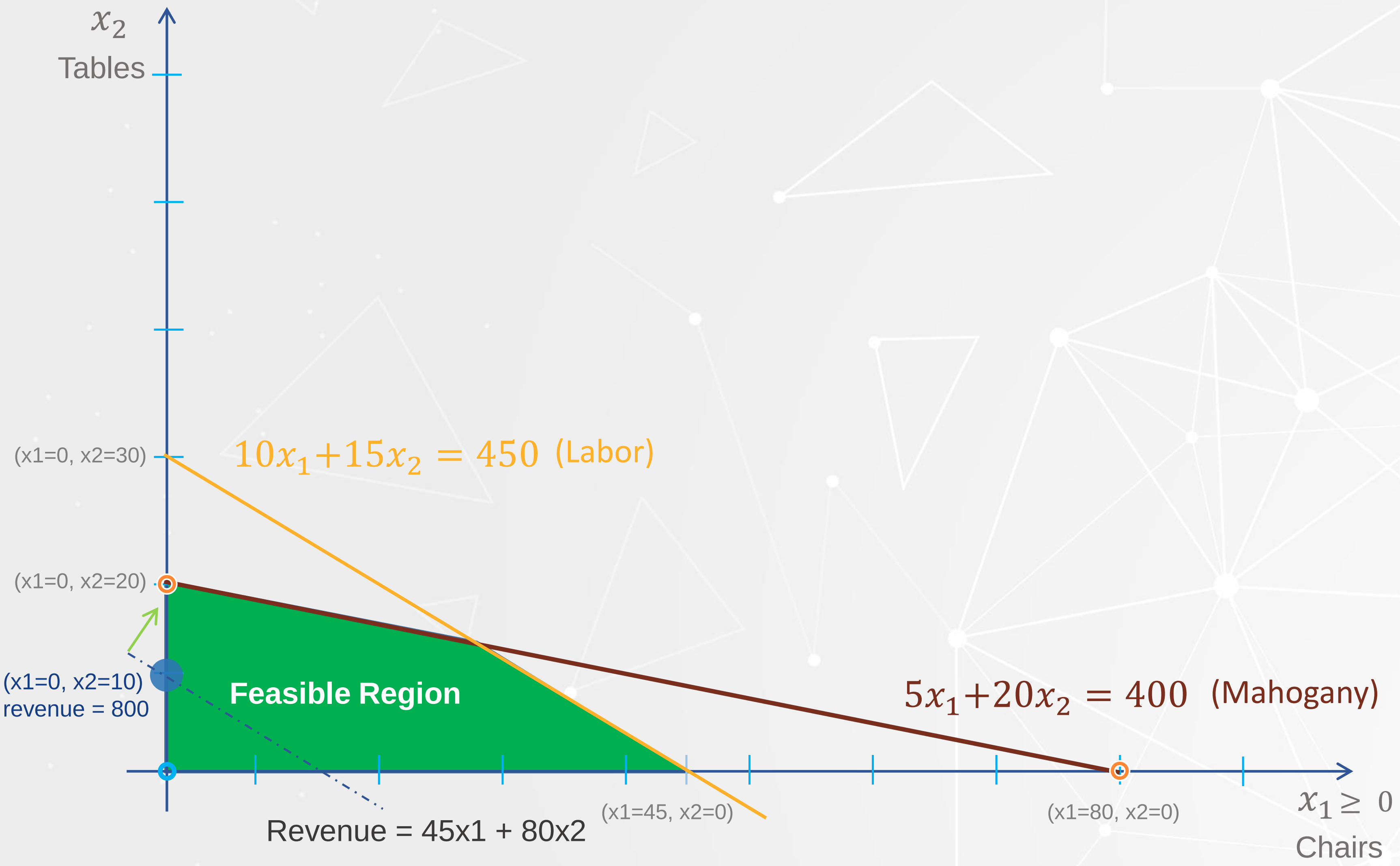
- If (x1=0, x2=0), then revenue is $0.00.

Graphical solution of Furniture Problem … 15



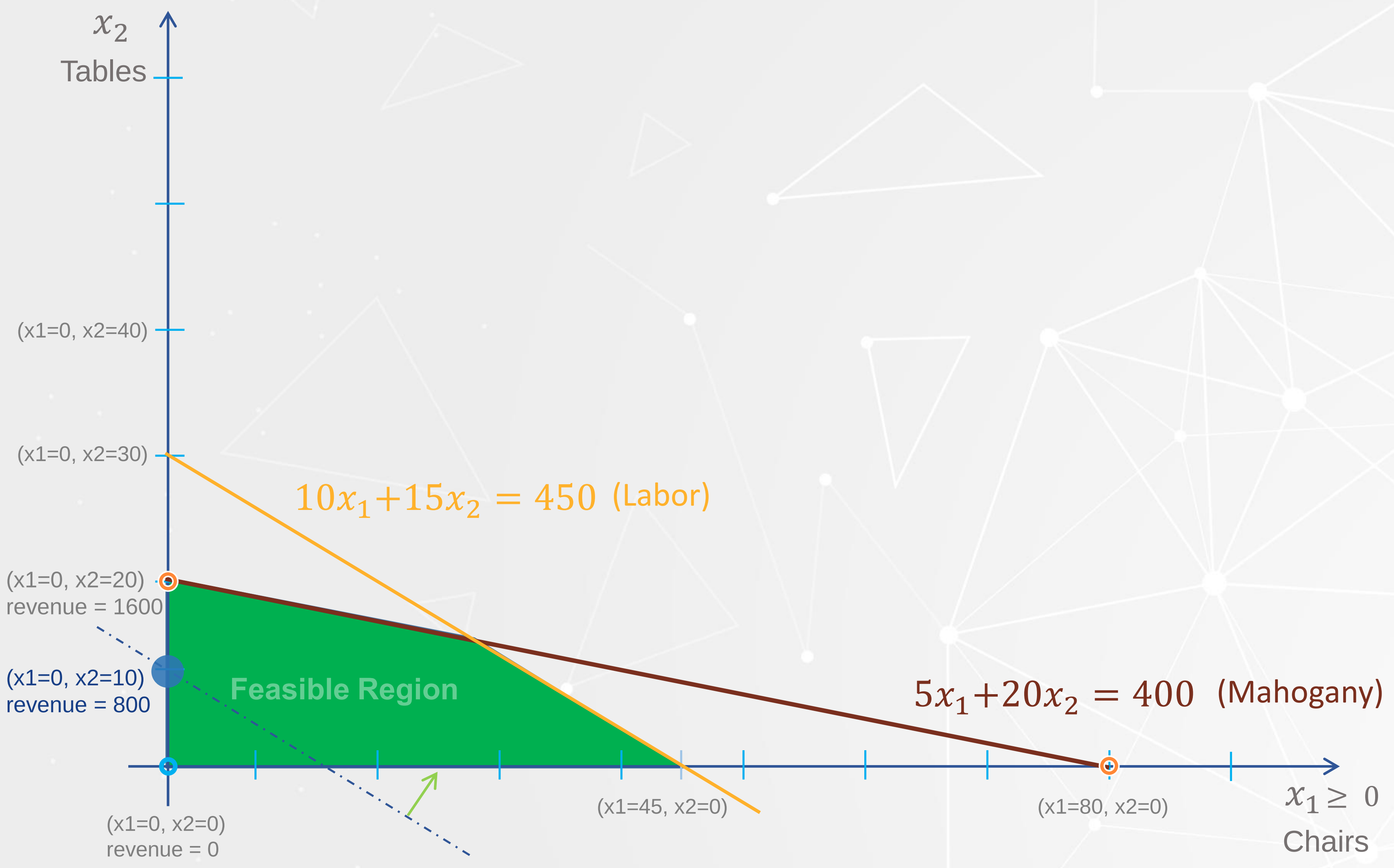
- Production Plan
  (x1 = 0, x2 = 10)
- Generates a revenue
  = 45(x1=0) + 80(x2=10)
  = $800

Graphical solution of Furniture Problem … 15



- Mahogany slack variable:
  h1 =
  400 – 5(x1=0) –
  20(x2=10)=200
- Labor slack variable:
  h2 =
  450 – 10(x1=0) –
  15(x2=10) = 150
- 200 units of unused mahogany capacity
- 150 units of unused labor capacity

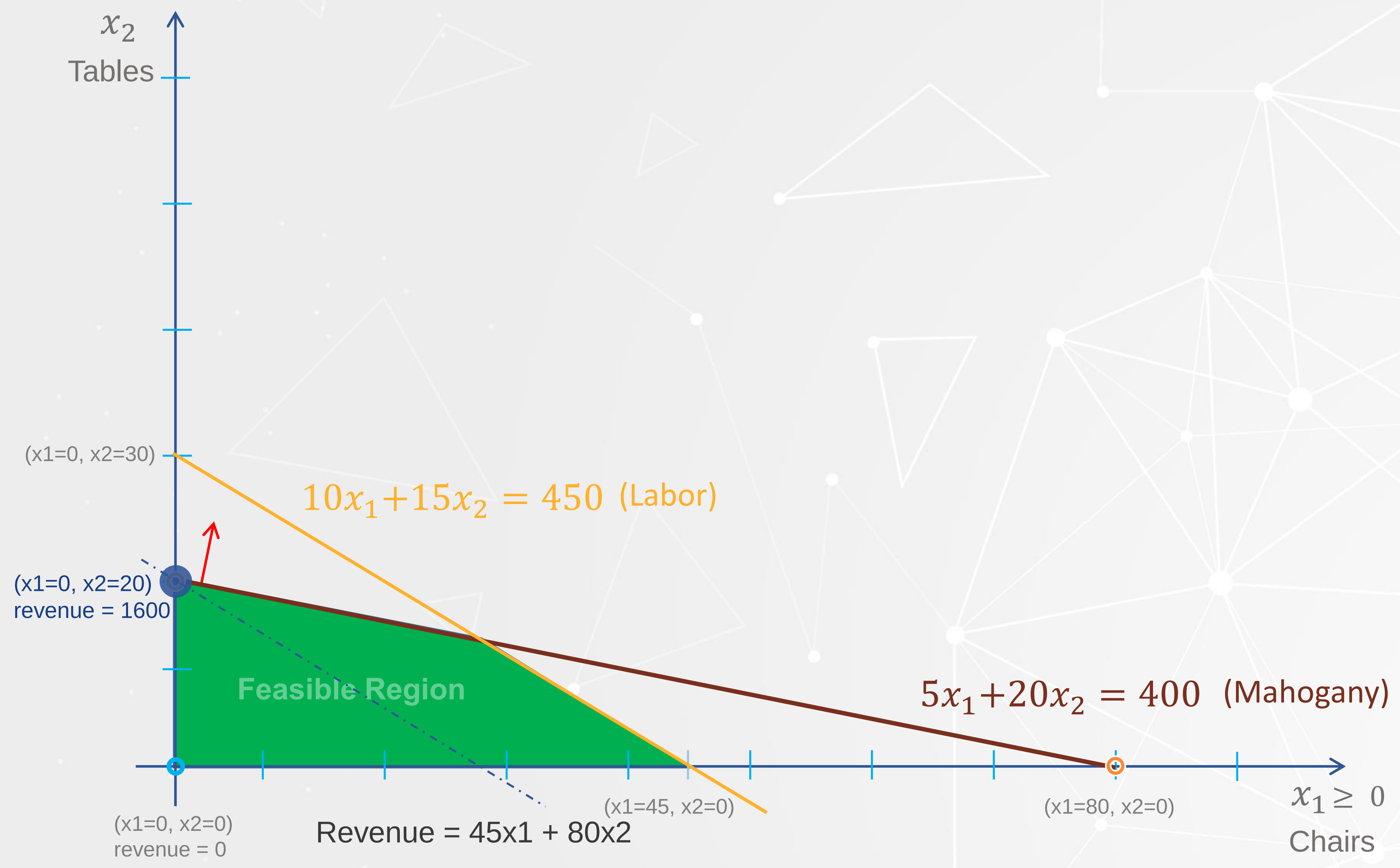
Graphical solution of Furniture Problem … 16



- How far can we increase the production of tables?
- Production Plan (x1=0, x2=20)

  Revenue = 45(x1 = 0) + 80(x2=20) = $1,600

Graphical solution of Furniture Problem … 16



- Can we continue increasing the production tables?

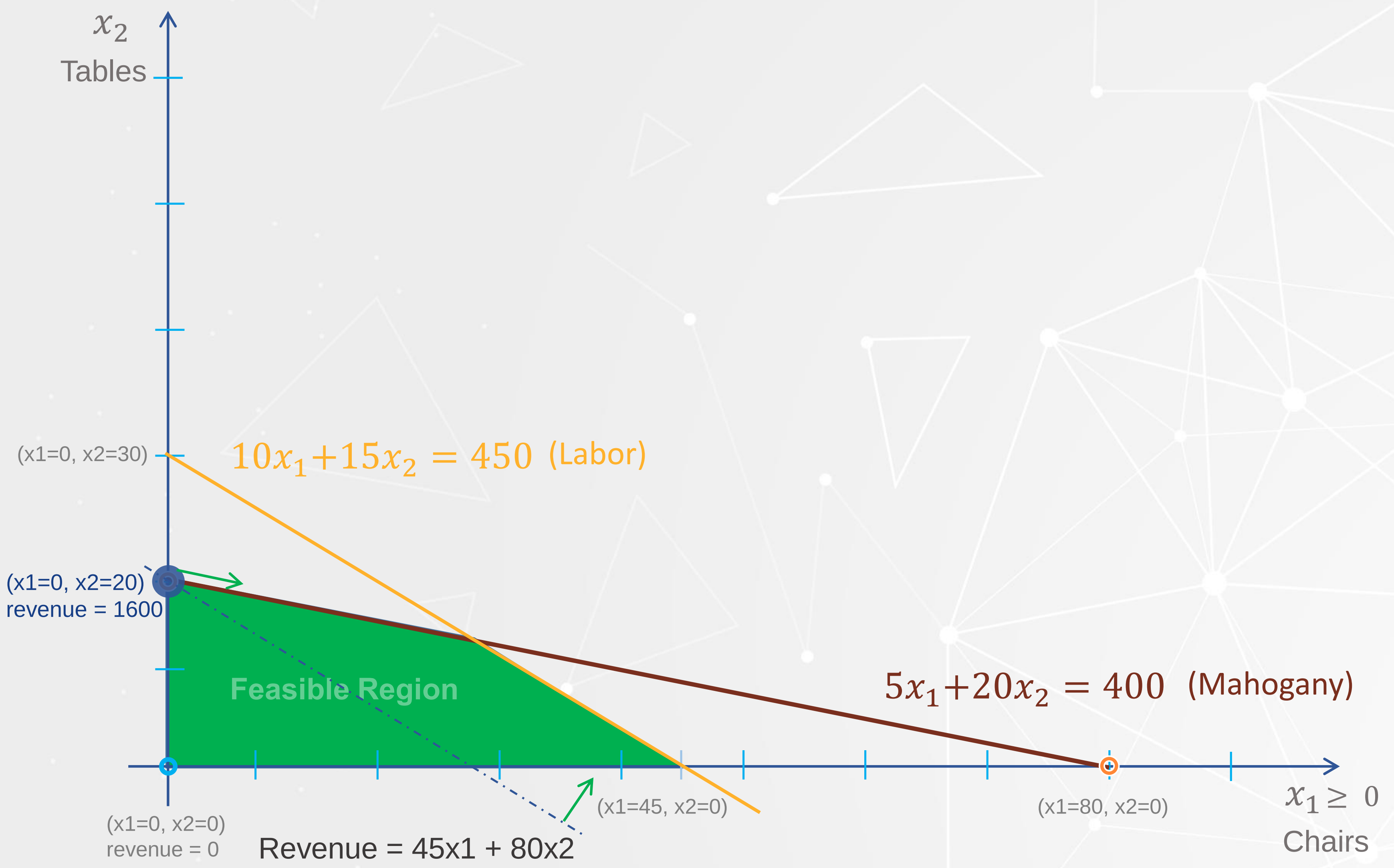
- Production plan (x1=0, x2=21)

  Revenue = 45(x1 = 0) + 80(x2=21) = $1,680

- Mahogany slack variable h1 = 400 – 5(x1=0) – 20(x2=21)= -20 !!!

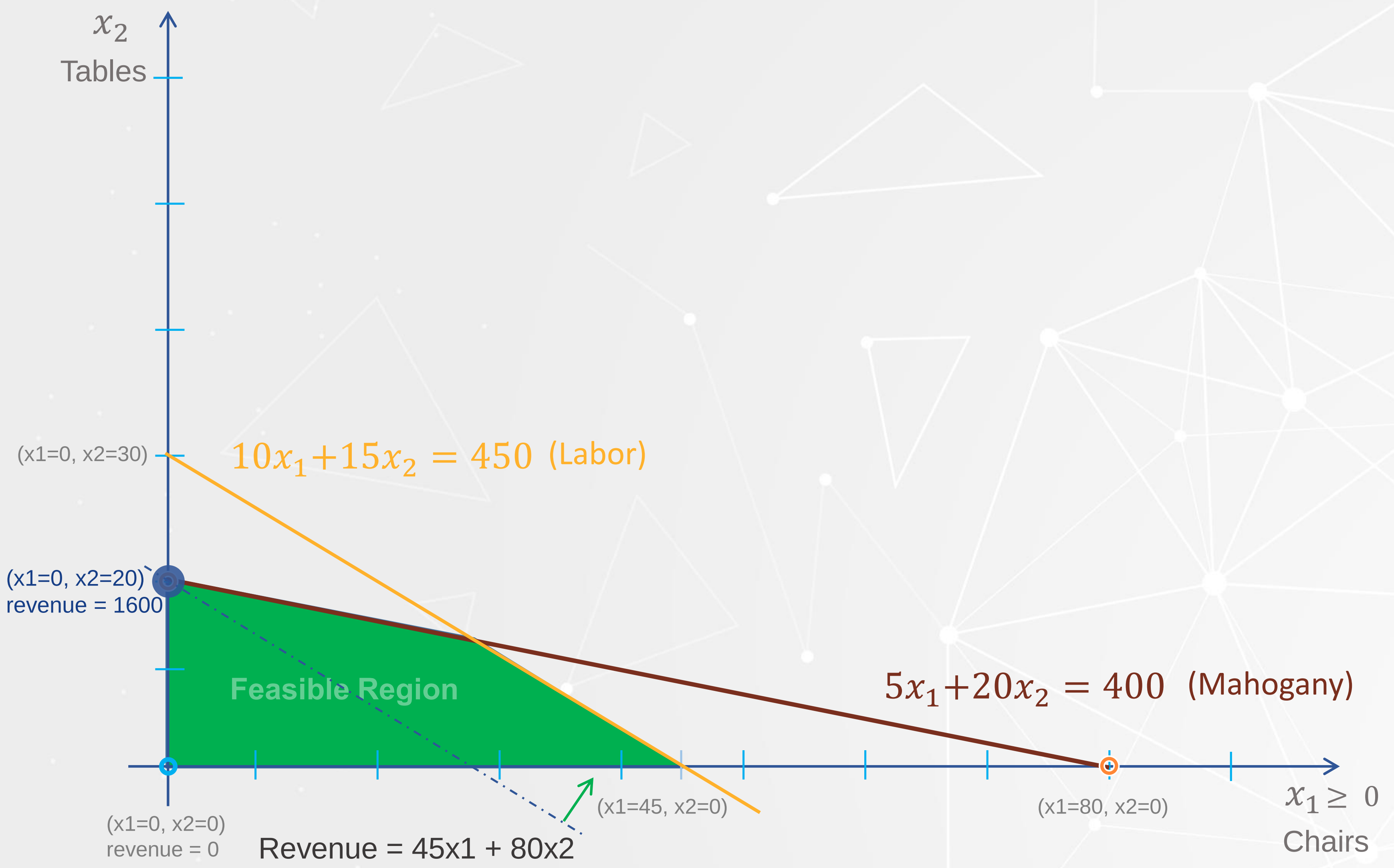
  Infeasible

Graphical solution of Furniture Problem … 17



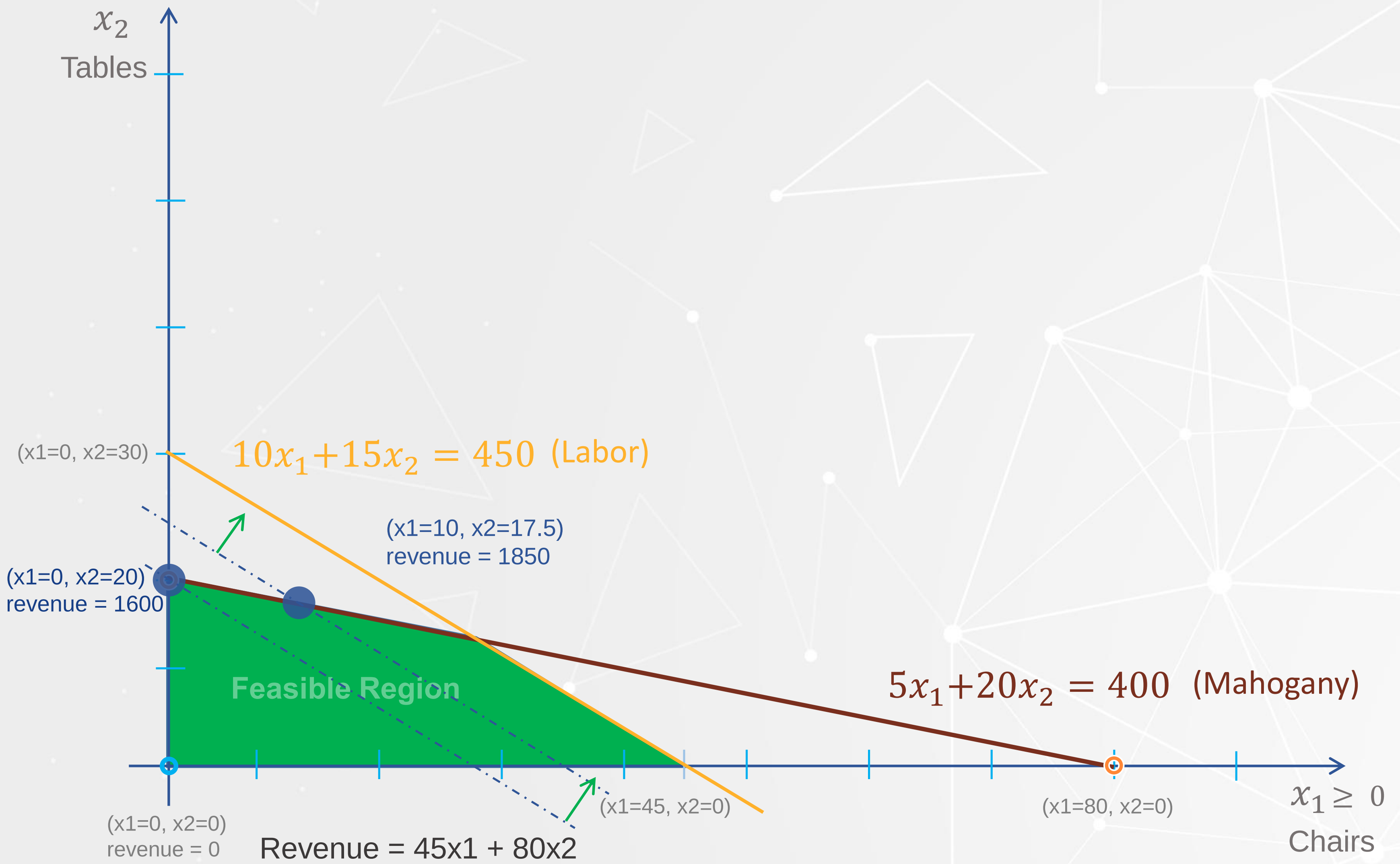
- What else we can do?

Graphical solution of Furniture Problem … 17



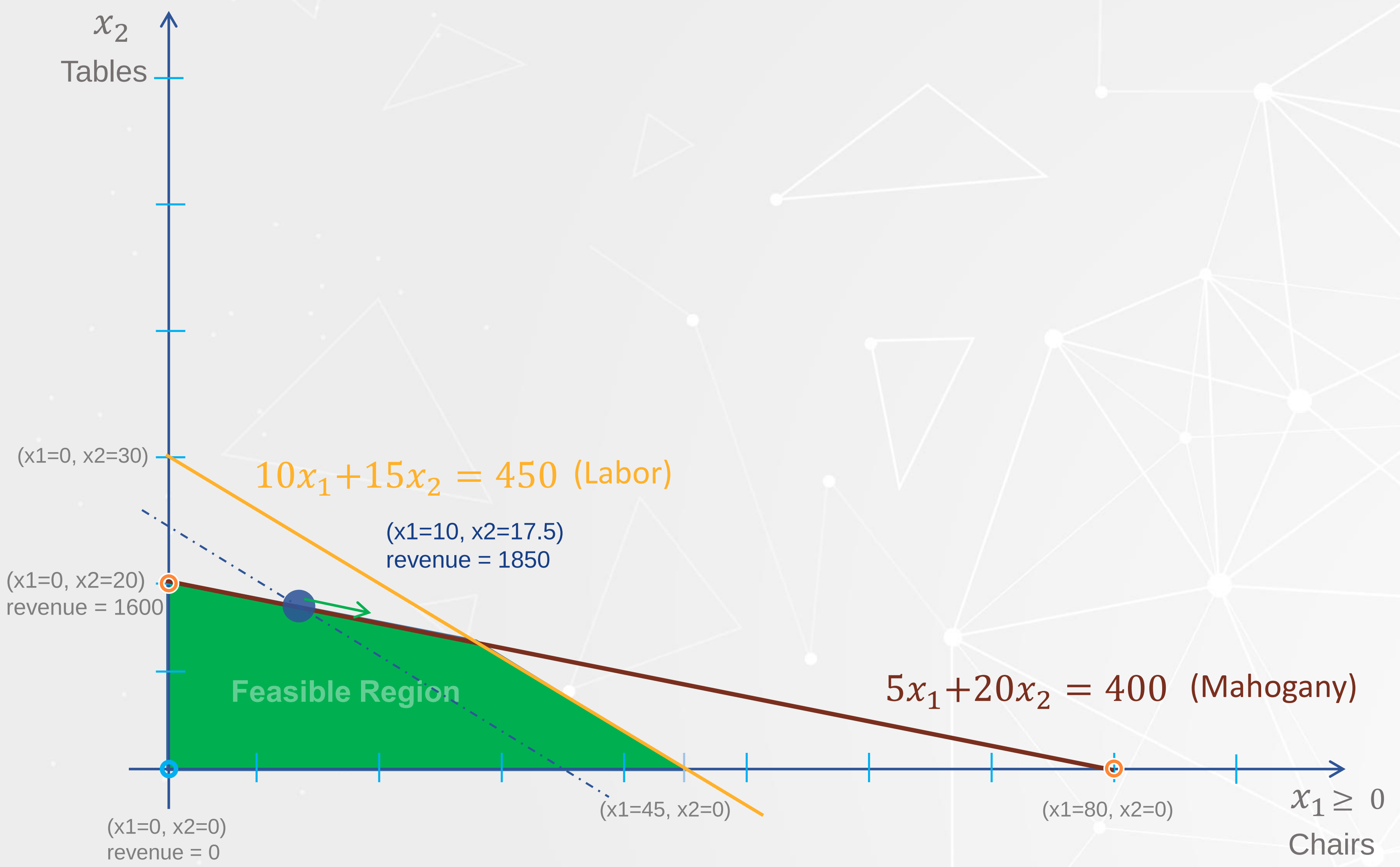
- Mahogany equation
  x2 =
  20 –(1/4)x1

Graphical solution of Furniture Problem … 18



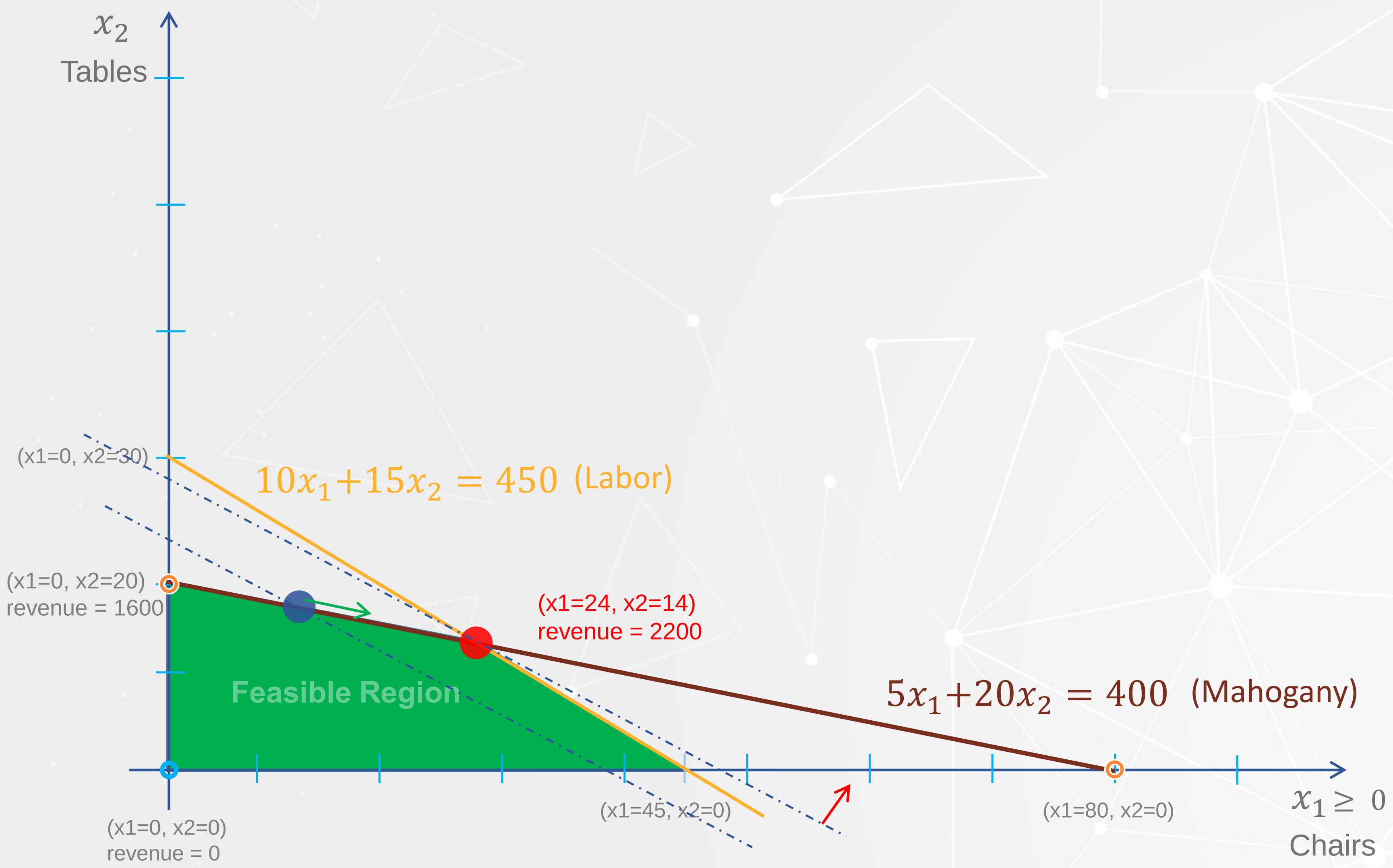
- How much can we keep increasing the production of chairs while keeping the production of tables as high as we can?
- If we build 10 chairs, then:
  x2 =
  20 –(1/4)(x1=10)
  = 17.5 tables.
- Mahogany slack variable
  h1 =
  400 – 5(x1=10) –
  20(x2=17.5) = 0

Graphical solution of Furniture Problem … 18



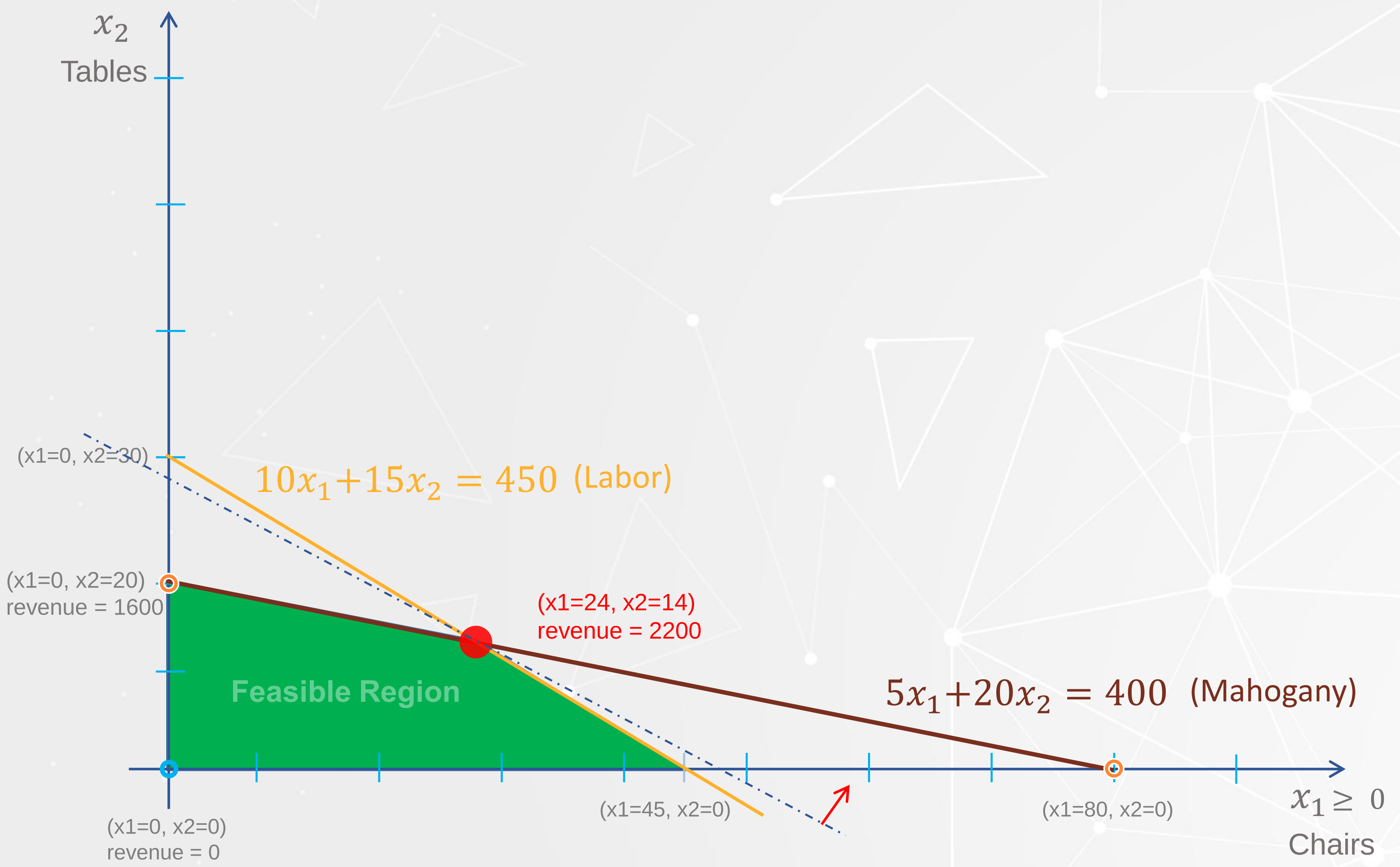
- Production plan (x1=10, x2=17.5)
- Revenue = 45(x1=10) + 80(x2=17.5) = $1,850
- Labor slack variable h2 = 450 – 10(x1=10) – 15(x2=17.5) = 87.5

Graphical solution of Furniture Problem … 19



- Observe that when the production plan moving along the mahogany equation hits the labor equation, we cannot move any further.
- This happens when the equation that defines the mahogany constraint intersects with the equation that defines the labor constraint. The associated production plan is found by solving these system of equations.
- **STOP**, labor constraint limits production plan.

Graphical solution of Furniture Problem … 19



Mahogany

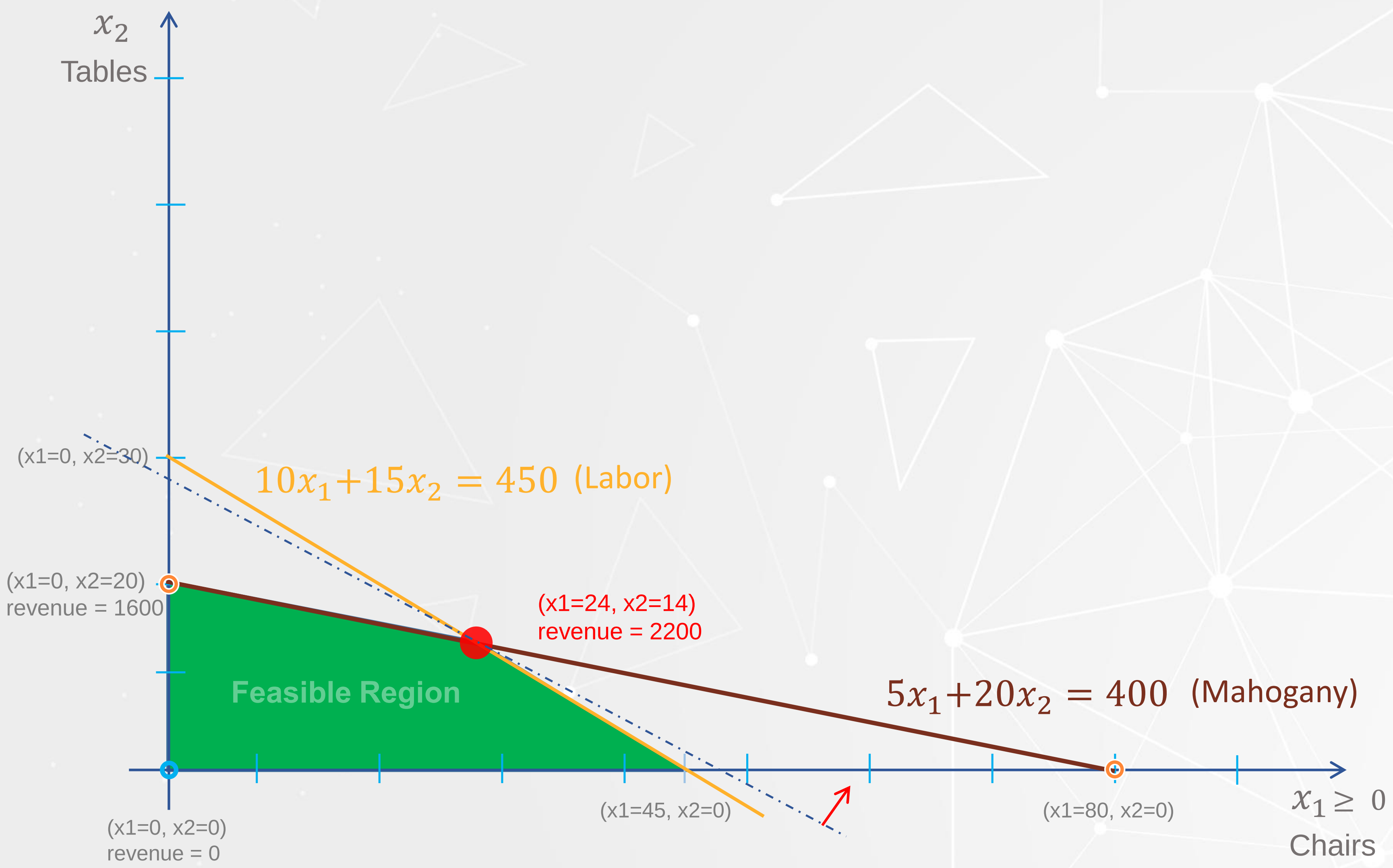
$$5x_1+20x_2 = 400$$

Labor

$$10x_1+15x_2 = 450$$

- Production Plan:
  - 24 chairs
  - 14 tables.
- Revenue = 45(x1=24) + 80(x2=14) = $2,200

Graphical solution of Furniture Problem … 19



Production Plan (x1 =24, x2=14) is optimal

Efficient Production Plan

Mahogany slack variable
h1 = 400 – 5(x1=24) – 20(x2=14)= 0

Labor slack variable
h2 = 450 – 10(x1=24) – 15(x2=14) = 0