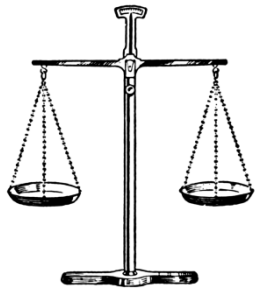

Modelling LP: An Inventory Problem

A Production Planning Problem: The Upton Corporation

The Upton Corporation manufactures heavy-duty air compressors for the home and light industrial markets. Upton is presently trying to plan its production and inventory levels for the next six months. Because of seasonal fluctuations in utility and raw material costs, the per unit cost of producing air compressors varies from month to month—as does the demand for air compressors. Production capacity also varies from month to month due to differences in the number of working days, vacations, and scheduled maintenance and training. The following table summarizes the monthly production costs, demands, and production capacity Upton's management expects to face over the next six months.

A Production Planning Problem: The Upton Corporation

- Upton is planning the production of their heavy-duty air compressors for the next 6 months.

	Month					
	1	2	3	4	5	6
Unit Production Cost	$240	$250	$265	$285	$280	$260
Units Demanded	1,000	4,500	6,000	5,500	3,500	4,000
Maximum Production	4,000	3,500	4,000	4,500	4,000	3,500
Minimum Production	2,000	1,750	2,000	2,250	2,000	1,750

A Production Planning Problem: The Upton Corporation

Given the size of Upton's warehouse, a maximum of 6,000 units can be held in inventory at the end of any month.

The owner of the company likes to keep at least 1,500 units in inventory as safety stock to meet unexpected demand contingencies.

To maintain a stable workforce, the company wants to produce no less than one half of its maximum production capacity each month.

Upton's controller estimates that the cost of carrying a unit in any given month is approximately equal to 1.5% of the unit production cost in the same month.

Upton estimates the number of units carried in inventory each month by averaging the beginning and ending inventory for each month.

There are 2,750 units currently in inventory.

Upton wants to identify the production and inventory plan for the next six months that will meet the expected demand each month while minimizing production and inventory costs.

A Production Planning Problem: The Upton Corporation

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Unit Production Cost	$240	$250	$265	$285	$280	$260
Units Demanded	1,000	4,500	6,000	5,500	3,500	4,000
Maximum Production	4,000	3,500	4,000	4,500	4,000	3,500
Minimum Production	2,000	1,750	2,000	2,250	2,000	1,750

- Beginning inventory = 2,750 units
- Safety stock = 1,500 units
- Unit carrying cost = 1.5% of unit production cost
- Maximum warehouse capacity = 6,000 units

Defining the Decision Variables

P_i = number of units to produce in month i, i=*1 to 6*

B_i = beginning inventory month i, i=*1 to 6*

Defining the Objective Function

Minimize the total cost production
& inventory costs.

$$\text{MIN: } 240P_1+250P_2+265P_3+285P_4+280P_5+260P_6$$

$$+ 3.6(B_1+B_2)/2 + 3.75(B_2+B_3)/2 + 3.98(B_3+B_4)/2$$

$$+ 4.28(B_4+B_5)/2 + 4.20(B_5+ B_6)/2 + 3.9(B_6+B_7)/2$$

Note: The beginning inventory in any month is the same as the ending inventory in the previous month.

Defining the Constraints - I

- Production levels

  $2{,}000 <= P_1 <= 4{,}000$ } month 1

  $1{,}750 <= P_2 <= 3{,}500$ } month 2

  $2{,}000 <= P_3 <= 4{,}000$ } month 3

  $2{,}250 <= P_4 <= 4{,}500$ } month 4

  $2{,}000 <= P_5 <= 4{,}000$ } month 5

  $1{,}750 <= P_6 <= 3{,}500$ } month 6

Defining the Constraints - II

- Ending Inventory (EI = BI + P - D)

$1{,}500 \leq B_1 + P_1 - 1{,}000 \leq 6{,}000$ } month 1

$1{,}500 \leq B_2 + P_2 - 4{,}500 \leq 6{,}000$ } month 2

$1{,}500 \leq B_3 + P_3 - 6{,}000 \leq 6{,}000$ } month 3

$1{,}500 \leq B_4 + P_4 - 5{,}500 \leq 6{,}000$ } month 4

$1{,}500 \leq B_5 + P_5 - 3{,}500 \leq 6{,}000$ } month 5

$1{,}500 \leq B_6 + P_6 - 4{,}000 \leq 6{,}000$ } month 6

Defining the Constraints - III

- Beginning Balances

$$B_1 = 2750$$

$$B_2 = B_1 + P_1 - 1{,}000$$

$$B_3 = B_2 + P_2 - 4{,}500$$

$$B_4 = B_3 + P_3 - 6{,}000$$

$$B_5 = B_4 + P_4 - 5{,}500$$

$$B_6 = B_5 + P_5 - 3{,}500$$

$$B_7 = B_6 + P_6 - 4{,}000$$

Optimal Solution

Upton Manufacturing

	Month					
	1	2	3	4	5	6
Beginning Inventory	2,750	5,750	4,750	2,750	1,500	2,000
Units Produced	4,000	3,500	4,000	4,250	4,000	3,500
Units Demanded	1,000	4,500	6,000	5,500	3,500	4,000
Ending Inventory	5,750	4,750	2,750	1,500	2,000	1,500
Minimum Production	2,000	1,750	2,000	2,250	2,000	1,750
Maximum Production	4,000	3,500	4,000	4,500	4,000	3,500
Minimum Inventory	1,500	1,500	1,500	1,500	1,500	1,500
Maximum Inventory	6,000	6,000	6,000	6,000	6,000	6,000
Unit Production Cost	$240	$250	$265	$285	$280	$260
Unit Carrying Cost 1.5%	$3.60	$3.75	$3.98	$4.28	$4.20	$3.90
Monthly Production Cost	$960,000	$875,000	$1,060,000	$1,211,250	$1,120,000	$910,000
Monthly Carrying Cost	$15,300	$19,688	$14,906	$9,084	$7,350	$6,825
					Total Cost	$6,209,403